Abel symposium, August 5th-10th 2007

Programme, last revised August 8th.

	Monday 6th	*Tuesday 7th*	*Wednesday 8th*	*Thursday 9th*
08.30 – 09.30	coffee+cake			
09.30 – 10.30	**F. Morel**	**S. Stolz**	**J. Lurie**	**N. Strickland**
	coffee+fruit	coffee+cake	coffee+fruit	coffee+cake
11.00 – 12.00	**M. Hopkins**	**A. Merkurjev**	**J. Baez**	**M. Levine**
	lunch	lunch	lunch	lunch
13.30 – 14.20	**R. Cohen**	**M. Behrens**	**H. Esnault**	**C. Rezk**
14.40 – 15.30	**L. Hesselholt**	**M. Rost**	**B. Toen**	**U. Jannsen**
	coffee+cake	excursion before conference dinner	coffee+cake	coffee+cake
16.30 – 17.20	**M. Ando**		**U. Tillmann**	**D. Freed**
17.40 – 18.30	**D. Sullivan**		**V. Voevodsky**	

Titles and abstracts

Matt Ando (UIUC):
"Units of ring spectra, and orientations"

The theory of units of associative and commutative ring spectra is analogous to the theory of the first Chern class in the classical theory of line bundles. Like the first Chern class, the units of a ring spectrum can usefully be approached from several points of view. I shall survey some of these approaches and an application to the string orientation of the spectrum of topological modular forms.

John Baez (UCR):
"Higher Gauge Theory and Elliptic Cohomology"

The concept of elliptic object suggests a relation between elliptic cohomology and `higher gauge theory', a generalization of gauge theory describing the parallel transport of strings. In higher gauge theory, we categorify familiar notions from gauge theory and consider `principal 2-bundles' with a given `structure 2-group'. These are a slight generalization of nonabelian gerbes. After a quick introduction to these ideas, we focus on the 2-group String_G coming from a compact simple Lie group G. We describe how this 2-group is built using a central extension of the loop group ΩG, and how the classifying space for String_G-2-bundles is related to the `string group' of elliptic cohomology. If there is time, we shall also describe a vector 2-bundle canonically associated to any principal 2-bundle, and how this relates to the von Neumann algebra construction of Stolz and Teichner.

Mark Behrens (MIT):
"Topological Automorphic Forms"

I will describe some cohomology theories of topological automorphic forms which arise from a theorem of Jacob Lurie. These are associated to Shimura varieties for the groups U(1,n-1) in the same way that topological modular forms are associated to the moduli space of elliptic curves. The corresponding theories detect v_n-periodic phenomena. I will also describe a related spectrum which approximates the K(n)-local sphere. This is joint work with Tyler Lawson.

Ralph Cohen (Stanford):
"Morse theory, Floer theory, and String topology"

In this talk I will describe the use of Morse theory to study string topology. This involves the use of ribbon graphs and the study of moduli spaces of gradient graph flows in the loop space of a manifold. I will then discuss the relationship between Morse theory on the loop space of a manifold with Floer theory on its cotangent bundle. I will review results of geometers such as C. Viterbo, D. Salamon and J. Weber, and A. Abbondandolo and M. Schwarz on this topic, and then describe a homotopy theoretic aspect to this subject. Namely, we show that that the Floer theory of the cotangent bundle can be "spectrified", by showing that the spaces of J-holomorphic cylinders in the cotangent bundle are framed manifolds that represent, via Pontrjagin-Thom theory, the attaching maps of a spectrum. We then show that this spectrum is stably equivalent to the suspension spectrum of the free loop space.

Hélène Esnault (Duisburg-Essen):
"Tate motives and fundamental group"

Let k be a number field, $S \subset P^1(k)$ a finite set. We relate the Deligne-Goncharov construction of the motivic fundamental group of P^1 - S to the Tannaka Q-group scheme of the category of mixed Tate motives over X. This work is joint with Marc Levine.

Dan Freed (Austin):
"Remarks on topological quantum field theory"

Lars Hesselholt (MIT/Nagoya):
"Algebraic K-theory and homeomorphisms of manifolds"

The celebrated work of Waldhausen, Weiss-Williams, and Farrell-Jones shows that the homotopy groups in low degrees of the space of homeomorphisms of a closed negatively curved Riemannian manifold of dimension $n \geq 5$ can be expressed as a functor of the fundamental group of the manifold. To determine this functor, however, it remains to determine the homotopy groups of the topological Whitehead space of the circle. In this talk, we evaluate the latter homotopy groups in low dimensions.

Mike Hopkins (Harvard):
“Structured ring spectra and homotopy theory”

This talk will be a general introduction to the notion of a structured ring spectrum and some of the roles it has played in homotopy theory. I plan to discuss some of the early applications of structured ring spectra in computations, the later use in the construction of new homology theories like tmf, and the deep, modern refinements of these ideas due to Jacob Lurie.

Uwe Jannsen (Regensburg):
"On finiteness results for motivic cohomology and resolution of singularities"

First we state that resolution of singularities implies some finiteness results for motivic cohomology of schemes of finite type over Z. This is partly joint work with Shuji Saito. Then we report on work in progress on embedded resolution of singularities for low-dimensional schemes. This is work in progress with V. Cossart and S. Saito.

Marc Levine (Northeastern):
"The motive of a Severi-Brauer variety and K-theory of central simple algebras"

This is joint work with Bruno Kahn. There are three pieces:

1. We compute the slices (in the sense of S^1-spectra) of K-theory with coefficients in an Azumaya algebra, arriving at a twisted version of the Atiyah-Hirzebruch spectral sequence relating motivic cohomology to K-theory.
2. We compute the slices of the motive of the Severi-Brauer variety associated to a prime degree central simple algebra and relate these to the slices in (1).
3. Putting (1) and (2) together with the Bloch-Kato conjecture gives some new results on the K-theory of a central simple algebra A of square free degree, for instance, that $SK_2(A)=0$.

Jacob Lurie (Harvard):
"The Baez-Dolan Cobordism Hypothesis in (Very) Low Dimensions"

One of the main sources of examples of higher categories is the theory of bordisms between manifolds. The Baez-Dolan cobordism hypothesis gives a (conjectural) characterization of these higher categories by a universal property. I will describe some variations on the cobordism hypothesis, focusing on manifolds of dimension at most two. I will then sketch a proof in dimension one, using techniques from the deformation theory of higher categories.

Alexander Merkurjev (UCLA):
"The Milnor and Bloch-Kato Conjectures"

A survey of the history and methods of the proofs of the Milnor and Bloch-Kato Conjectures will be presented. A computation of the motivic cohomology of the generalized Rost motive will be given. This is joint work with Andrei Suslin.

Fabien Morel (München):
"Unstable A^1-Homotopy Theory"

In this talk we will survey known computations of A^1-homotopy or homology groups/sheaves in unstable A^1-homotopy theory. We will emphasize problems involving the π_0 and π_1, especially concerning ``geometric classifying spaces" and ``A^1-connected projective smooth varieties"; in the latter case the π_1 should play a fundamental role, as the usual π_1 plays in the classical surgerical approach.

Charles Rezk (UIUC):
"Colinear approximations to homotopy theory"

Markus Rost (Bielefeld):
"The basic correspondence of a norm variety"

The basic correspondence is a major tool to prove genericity of norm varieties and to split off its motive. We will discuss some details and its role in the proof of the Bloch-Kato conjecture (bijectivity of the norm residue homomorphism).

Stephan Stolz (Notre Dame):
"Quantum Field Theories and Generalized Cohomology"

This is a talk on an ongoing joint project with Peter Teichner (Berkeley). Based on Graeme Segal's axiomatization of quantum field theories, we incorporate super symmetry into the picture and define the notion of a super symmetric d-dimensional field theory over a manifold X for d=0,1,2. It turns out that 0-dimensional QFT's over X can be identified with closed differential forms, and a vector bundle with connection leads to a 1-dimensional QFT over X, while 2-dimensional QFT's over X seem to be genuinely new geometric objects that don't have a classical description.

The topological information these objects carry is revealed by passing to concordance classes of QFT's over X, where we call two QFT's concordant if there is a QFT over the product of X with the interval that restricts to the given QFT's on the boundary. The above description of 0-dimensional QFT's implies that their concordance classes can be identified with ordinary cohomology with real coefficients; we can show that concordance classes of d-dimensional QFT's gives K-theory for d=1 and conjecture that we obtain elliptic cohomology for d=2 (more precisely, the `topological modular form theory' of Hopkins-Miller). Evidence for the latter is provided by our result that we can show that the `partition function' of a 2-dimensional QFT is an integral modular function.

Neil Strickland (Sheffield):
"Symmetric powers of spheres"

This talk will report on a project to understand, extend and consolidate a dense network of connections between a wide range of ideas in stable homotopy theory.

One way into the maze is to consider the symmetric powers of the sphere spectrum, which interpolate between the sphere spectrum itself and the integer Eilenberg-Mac Lane spectrum. The quotients in this filtration are interesting spectra that arise naturally

in a number of other contexts, involving the theory of Steinberg modules and Hecke algebras and the combinatorics of partition complexes. The same partition complexes are also relevant in the theory of the Goodwillie tower of the identity functor. There are other connections with power operations in Morava E-theory, as well as the classical Dyer-Lashof algebra and Lambda algebra. A great deal is already known about these ideas, but there are some hints that important parts of the puzzle have yet to fall into place.

Dennis Sullivan (SUNY):
"String Topology Update"

The algebraic topology of a manifold, in particular its Poincaré duality, implies a rich algebraic structure in the topology of the free loop space of the manifold. At one level there are chain operations parametrized by the chains on the open moduli space of Riemann surfaces (compare Ralph Cohen's talk). At the next level it is possible to describe what happens to these operations as one approaches infinity of moduli space. The chain operations at infinity actually factor, or they can be filled in by chain homotopies to zero.

Ulrike Tillmann (Oxford):
"CFT, Mumford's conjecture, and slices of Thom's cobordism theory"

In joint work with Ib Madsen, Søren Galatius and Michael Weiss, the classifying space of cobordism categories has been identified with certain (infinite loop) spaces associated to Thom spaces. In this lecture, I will explain this theorem, some extensions, and its connection to CFT and Mumford's conjecture.

Bertrand Toën (Toulouse):
"Loop spaces, Chern character and derived algebraic geometry"

For any scheme X we construct a derived scheme LX, called the "derived loop space of X", which is the derived scheme of maps from the simplicial circle to X. In a first part I will explain the relations between the theory of functions on LX and cyclic homology of X, which will then be used in order to give a construction of the Chern character. In a second part I will explain how this can be generalized in order to construct a "secondary Chern character" defined for certain "sheaves of derived categories" on X (rather than sheaves of vector spaces) and with values in a "secondary cyclic homology". The domain of this secondary Chern character is the Grothendieck group of sheaves of derived categories on X, which is possibly related to (a part of) elliptic cohomology of X.

Vladimir Voevodsky (IAS):
"Symmetric powers of motives"