Flows on a separable C^*-algebra

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1 Flows

By a flow α on a C^*-algebra A we mean a homomorphism $\alpha : \mathbf{R} \to \mathrm{Aut}(A)$ such that $t \mapsto \alpha_t(x)$ is continuous for each $x \in A$, where $\mathrm{Aut}(A)$ is the automorphism group of A. When α is a flow, we denote by δ_α the generator of α, which is a closed derivation in A, i.e., δ_α is a closed linear map defined on a dense $*$-subalgebra $\mathcal{D}(\delta_\alpha)$ of A into A such that $\delta_\alpha(x)^* = \delta_\alpha(x^*)$ and

$$\delta_\alpha(xy) = \delta_\alpha(x)y + x\delta_\alpha(y)$$

for $x, y \in \mathcal{D}(\delta_\alpha)$. See [3, 4, 1, 23] for characterizations of generators and more.

Given $h \in A_{sa}$, $\delta_\alpha + \mathrm{ad}\, ih$ is again a generator. We denote by $\alpha^{(h)}$ the flow generated by $\delta_\alpha + \mathrm{ad}\, ih$. We call $\alpha^{(h)}$ an inner perturbation of α. More generally, if u is an α-cocycle, i.e., $u : \mathbf{R} \to \mathcal{U}(A)$ is continuous such that $u_s\alpha_s(u_t) = u_{s+t},\ s, t \in \mathbf{R}$, then $t \mapsto \mathrm{Ad}\, u_t\alpha_t$ is a flow, called a cocycle perturbation of α. Note that an inner perturbation is a cocycle perturbation; $\alpha^{(h)}$ is obtained as $\mathrm{Ad}\, u\alpha$, where u is the (differentiable) α-cocycle defined by $du_t/dt = u_t\alpha_t(ih)$. In general a cocycle perturbation of α is given as $t \mapsto \mathrm{Ad}\, v\alpha_t^{(h)}\mathrm{Ad}\, v^*$ for some $v \in \mathcal{U}(A)$ and $h \in A_{sa}$.

A (non-degenerate) representation π of the system (A, α) is called covariant if there is a unitary flow U on the Hilbert space $\mathcal{H}_\pi$ such that $\mathrm{Ad}\, U_t\pi = \pi\alpha_t,\ t \in \mathbf{R}$. In general we do not seem to know a good characterization for existence of covariant irreducible representations, but in the following discussions this will be a standing assumption. (There is an obstruction for the flow α to have a faithful family of covariant irreducible representations, i.e., for some $t > 0$ there is a $u \in \mathcal{U}(A)$ such that $\alpha_t = \mathrm{Ad}\, u$ and u is not left invariant under α.)

The following is an adaptation of Glimm's theorem (see [21] for the Connes spectrum).

Theorem 1.1 *[12] Let A be a separable prime C^*-algebra and let α be a flow on A with the Connes spectrum $\mathbf{R}(\alpha) \neq \{0\}$. Then the following conditions are equivalent:*

1. *There exists a faithful family of covariant irreducible representations of (A, α).*
2. *There exists a faithful covariant irreducible representation of (A, α) which induces a representation of the crossed product $A \times_\alpha \mathbf{R}$ (on $\mathcal{H}_\pi$), whose kernel is left invariant under $\hat{\alpha}|\mathbf{R}(\alpha)$.*
3. *For any UHF algebra D and any UHF flow γ on D (i.e., $\gamma_t = \bigotimes_{n=1}^\infty \mathrm{Ad}\, e^{ith_n}$ on $D = \bigotimes_{n=1}^\infty M_{k_n}$ with $h_n = h_n^* \in M_{k_n}$) such that $\mathrm{Sp}(\gamma) \subset \mathbf{R}(\alpha)$, and $\epsilon > 0$, there is a C^*-subalgebra B of A, an $h \in A_{sa}$, and a closed projection $q \in A^{**}$ such that*

$$\begin{array}{rcl} \|h\| & < & \epsilon, \\ \alpha_t^{(h)}(B) & = & B, \\ (\alpha_t^{(h)})^{**}(q) & = & q, \\ qAq & = & Bq, \\ (Bq, \alpha^{(h)}|Bq) & \cong & (D, \gamma), \end{array}$$

and if $c(q)$ denotes the central support of q in A^{**}, $x = 0$ iff $xc(q) = 0$ for any $x \in A$.

There is a similar result in the case $\mathbf{R}(\alpha) = \{0\}$, where the condition 3 should be modified. Thus the above result tells us that if there is a faithful family of covariant irreducible representations, then there are all sorts of covariant representations (via such representations of UHF flows).

Given a flow α we denote by G_α the group of automorphisms γ with the property that $\gamma\alpha\gamma^{-1}$ is a cocycle perturbation of α, which is regarded as a largest possible symmetry group for α.

In [20] it is shown that if π_1 and π_2 are irreducible representations of A with $\mathrm{Ker}(\pi_1) = \mathrm{Ker}(\pi_2)$, then there is an asymptotically inner automorphism γ such that $\pi_1\gamma$ is equivalent to π_2. The following is an adaptation to covariant irreducible representations.

Theorem 1.2 *[18] Let A be a separable prime C^*-algebra and let α be a flow on A with non-trivial Connes spectrum. If (π_1, U_1) and (π_2, U_2) are covariant irreducible representations of (A, α) such that $\mathrm{Ker}(\pi_1 \times U_1) = \mathrm{Ker}(\pi_2 \times U_2)$, then there is a $\gamma \in G_\alpha$ such that $\pi_1\gamma$ is equivalent to π_2.*

The above γ is actually asymptotically inner and extends to an automorphism $\overline{\gamma}$ of the crossed product $A \times_\alpha \mathbf{R}$ such that $(\pi_1 \times U_1)\overline{\gamma} = \pi_2 \times U_2$.

2 Approximately inner flows

If the C^*-algebra A is not commutative, there is always a non-trivial flow; choose a self-adjoint $h \in A$ such that $h \notin A \cap A'$ and set $\alpha_t = \mathrm{Ad}\, e^{ith}$, which defines an inner flow α.

If (α^n) is a sequence of flows and α is a flow such that for every $x \in A$ $(\alpha_t^n(x))$ converges to $\alpha_t(x)$, uniformly in t on every bounded set, as $n \to \infty$, we say that (α^n) converges to α. (This convergence can be expressed in terms of generators; δ_α is the graph limit of (δ_{α^n}).) If all α^n's are inner, we call α approximately inner (or AI). If α is uniformly continuous, then it is known that α is AI. Furthermore if α is almost uniformly continuous (or α^* is strongly continuous on A^*), then α is AI. The class of AI flows is certainly much wider.

Proposition 2.1 *Let A be a C^*-algebra of real rank zero and α an AI flow. Then there exists a faithful family of covariant irreducible representations for (A, α).*

The following shows the existence of non-trivial AI flows.

Theorem 2.2 *[13] Let A be a separable antiliminary C^*-algebra. Then there is a AI flow α on A whose Connes spectrum is full. Moreover if S is a countable set of irreducible representations of A, there is such a flow α with the property that α is covariant in every $\pi \in S$. (We may replace the condition of full Connes spectrum by the property that for any non-empty open set $O \subset \mathbf{R}$ there is a central sequence (z_n) in the spectral subspace $A^\alpha(O)$ such that $\lim_n \|xz_n\| = 0$ implies $x = 0$ for any $x \in A$.)*

We prove this as follows. We fix a sequence (π_i) of irreducible representations of A such that $\bigcap \mathrm{Ker}(\pi_i) = \{0\}$. We will define α as

$$\alpha_t = \lim_n \mathrm{Ad}\, e^{it(h_1 + h_2 + \cdots + h_n)}$$

where a sequence (h_n) in A_{sa} is bounded and sufficiently central such that $\pi_i(e^{it(h_1+\cdots+h_n)})$ converges strongly as $n \to \infty$. We construct such (h_n) so that there is a central sequence (w_n) with $\alpha_t(w_n) \approx e^{itp_n} w_n$ for a prescribed (p_n) and $(\pi_i(w_n))$ is non-trivial for each i (w_n would almost commute with h_m, $m \neq n$). For the construction of the central sequences we use the main idea of [20].

If α is an (non uniformly continuous) AI flow on A, there are at least two equivalent classes of covariant irreducible representations; representations corresponding to ground states and ceiling states. We do not know if there are infinitely many or not in general. On the other hand we know that there are uncountably many equivalent classes of irreducible representations which are not covariant.

Theorem 2.3 *[13] Let α be an AI flow on a separable C^*-algebra A and let (π, U) be a covariant type I representation of (A, α) on a separable Hilbert space, i.e., $\mathrm{Ad}\, U_t \pi = \pi\alpha_t$. Then there is a sequence (h_n) in A_{sa} such that $(\mathrm{Ad}\, e^{ith_n})$ converges to α and $e^{it\pi(h_n)}$ strongly converges to U_t uniformly in t on every bounded set.*

This is not entirely trivial. With (h_n) as given in the definition of AI, we have to replace it in general by $(u_n(h_n + z_n)u_n^*)$ to meet the required condition, where (z_n) is a sequence in A_{sa} sufficiently central and (u_n) is a sequence in $\mathcal{U}(A)$ with $\|u_n - 1\| \to 0$.

3 AF flows

If A is an AF algebra and a flow α on A has an increasing sequence (A_n) of α-invariant finite-dimensional C*-subalgebras of A with dense union, then we call α an AF flow. In this case since there is an $h_n \in (A_n)_{sa}$ such that $\alpha_t|A_n = \mathrm{Ad}\, e^{ith_n}|A_n$, α is AI. There are example of non-AI flows on some AF algebra [14] (cf. [22]).

An AF flow can be constructed in an inductive way. The set of KMS states for an AF flow is calculable in a sense and seems to be full of variety [23, 10].

When B and C are C*-subalgebras of A, we write $B \overset{\delta}{\subset} C$ if for any $x \in B$ there is $y \in C$ such that $\|x - y\| \leq \delta\|x\|$. We define the distance of B and C by

$$\mathrm{dist}(B, C) = \inf\{\delta > 0 \mid B \overset{\delta}{\subset} C,\ C \overset{\delta}{\subset} B\}.$$

If α is an AF flow, then a cocycle perturbation α' of α may not be an AF flow but an *approximate AF flow* in the sense that $\sup_{t\in[0,1]} \mathrm{dist}(\alpha'_t(A_n), A_n) \to 0$, where the sequence (A_n) is chosen for α as above.

Theorem 3.1 *Let α be a flow on a unital AF algebra A. Then the following conditions are equivalent:*

1. *α is a cocycle perturbation of an AF flow.*
2. *α is an approximate AF flow, i.e., there is an increasing sequence (A_n) of finite-dimensional C*-subalgebras of A such that $\bigcup_n A_n$ is dense in A and*

$$\sup_{t\in[0,1]} \mathrm{dist}(\alpha_t(A_n), A_n) \to 0$$

as $n \to \infty$.

The proof of this result must use a result of Christensen [5].

For a C^*-algebra A we denote by $\ell^\infty(A)$ the C^*-algebra of bounded sequences in A and by $c_0(A)$ the ideal of $\ell^\infty(A)$ consisting of $x = (x_n)$ for which $\lim_{n\to\infty} \|x_n\| = 0$ and let $A^\infty = \ell^\infty(A)/c_0(A)$. We embed A into A^∞ by regarding each $x \in A$ as the constant sequence $(x, x, \ldots)$. Given a flow α on A we denote by $\ell^\infty_\alpha(A)$ the C*-algebra of $x = (x_n) \in \ell^\infty(A)$ for which $t \mapsto \alpha_t(x) = (\alpha_t(x_n))$ is norm-continuous and define A^∞_α as its image in A^∞. We naturally have the flow $\overline{\alpha}$ on A^∞_α induced by α. We will also denote by α the restriction of $\overline{\alpha}$ to $A' \cap A^\infty_\alpha$.

The following properties shared by AF flows and their cocycle perturbations could be used to distinguish them from other flows [11, 2].

Proposition 3.2 *Let A be a unital AF algebra and let α be a cocycle perturbation of an AF flow on A. Then $(A^\infty_\alpha \cap A')^\alpha$ has real rank zero and has trivial K_1. Moreover $(A^\infty_\alpha)^\alpha$ has real rank zero and trivial K_1.*

There are some examples of AI flows on an AF algebra without the above types of properties (see Section 3 of [11] and 3.11 of [2], where only the properties for $(A_\alpha^\infty)^\alpha$ are explicitly mentioned). Those examples are of the following type. Let C be a maximal abelian C*-subalgebra (masa) of A and choose a sequence (h_n) in C_{sa} such that the graph limit δ of $(\mathrm{ad}\, ih_n)$ is densely-defined and hence generates a flow [3, 23]. This is what we have as the examples and might be called a *quasi AF flow*. Note that the domain $\mathcal{D}(\delta)$ of δ contains the masa C (which is actually a Cartan masa in our examples); but depending on (h_n) it may contain another masa. (We know of no example of a generator whose domain does not contain a masa.)

Theorem 3.3 *Let α be an AF flow on a unital simple AF algebra A. Then α is a cocycle perturbation of an AF flow if and only if the domain $\mathcal{D}(\delta_\alpha)$ contains a canonical AF masa of A, where C is a canonical AF masa if there is an increasing sequence (A_n) of finite-dimensional C*-subalgebras of A with dense union such that C is the closure of $\bigcup_n C\cap A_n\cap A'_{n-1}$ with $A_0=0$.*

We note the following uniqueness result for canonical AF masas.

Proposition 3.4 *The canonical AF masas of an AF algebra are unique up to automorphism, i.e., if A be an AF algebra and C_1 and C_2 are canonical AF masas of A, then there is an automorphism ϕ of A such that $\phi(C_1)=C_2$.*

4 Rohlin flows

So far we have dealt with approximately inner (AI) flows. Here we deal with an extreme opposite.

Definition 4.1 *Let A be a C^*-algebra and α a flow on A. The flow α is said to have the* Rohlin property *if for any $p\in\mathbf{R}$ there is a sequence (u_n) in $\mathcal{U}(M(A))$ such that $\|\alpha_t(u_n)-e^{ipt}u_n\|\to 0$ uniformly in t on every compact subset of $\mathbf{R}$ and $\|[u_n,x]\|\to 0$ for any $x\in A$.*

In the case A is unital and hence $M(A)=A$, we have the following consequence:

Lemma 4.2 *[8] Let α be a Rohlin flow on a unital C^*-algebra A and $L>0$. Let γ denote the flow on $C(\mathbf{R}/L\mathbf{Z})$ by translations. There exists a sequence (ϕ_n) of linear maps of $A\otimes C(\mathbf{R}/L\mathbf{Z})$ into A such that $\phi_n(a\otimes 1)=a,\ a\in A$, (ϕ_n) is an approximate homomorphism, and $\|\phi_n(\alpha_t\otimes\gamma_t)(x)-\alpha_t\phi_n(x)\|\to 0$ for $x\in A\otimes C(\mathbf{R}/L\mathbf{Z})$.*

By using this fact we show that if α has the Rohlin property, then it has the one-cocycle property; i.e., if $u:\mathbf{R}\to\mathcal{U}(A'\cap A_\alpha^\infty)$ is an α-cocycle, then there is a $v\in\mathcal{U}(A'\cap A_\alpha^\infty)$ such that $u_t=v\alpha_t(v^*)$. This is the property we actually need. If A is non-unital, we still use the above lemma. This means we have to find out an approximate unit (e_n) consisting of projections such that $\delta_\alpha(e_n)\to 0$; we can achieve this only for special cases.

Examples of Rohlin flows are given for unital simple AT algebras of real rank zero [9] and also for Cuntz algebras [15, 19]. (But there are no Rohlin flows for unital AF algebras.)

Let $\mathcal{O}_n$ denote the Cuntz algebra generated by n isometries $s_1,\ldots,s_n$ with $2\le n<\infty$. Given a finite sequence $(p_1,\ldots,p_n)$ in $\mathbf{R}$ we define a flow α, called a quasi-free flow, on $\mathcal{O}_n$ by

$$\alpha_t(s_k)=e^{ip_kt}s_k,\quad k=1,2,\ldots,n.$$

Note that then δ_α vanishes on the masa C_n of $\mathcal{O}_n$ generated by $s_{i_1}s_{i_2}\cdots s_{i_m}s_{i_m}^*\cdots s_{i_1}^*$ for all finite sequences $(i_1,i_2,\ldots,i_m)$ and that C_n is a Cartan AF masa of $\mathcal{O}_n$.

Theorem 4.3 *Let α be the quasi-free flow corresponding to $p_1,\ldots,p_n$ as above. Then the following conditions are equivalent:*

1. α *has the Rohlin property.*

2. *The crossed product* $\mathcal{O}_n \times_\alpha \mathbf{R}$ *is purely infinite and simple.*

3. $p_1, \ldots, p_n$ *generate* $\mathbf{R}$ *as a closed subsemigroup.*

The hardest part of the proof lies in (3)⇒(1). When (3) is satisfied, we show for any $p \in \mathbf{R}$ there is a sequence (u_n) in $\mathcal{U}(\mathcal{O}_n)$ such that $\|\alpha_t(u_n) - e^{ipt}u_n\| \to 0$. This can be shown by combinatorial arguments. To find a central sequence with this property, we find a sequence (ϕ_k) of endomorphisms of $\mathcal{O}_n$ such that $\phi_k\alpha_t = \alpha_t\phi_k$ and $(\phi_k(x))$ belongs to $A' \cap A^\infty_\alpha$ for $x \in A$. (As a matter of fact ϕ_k is an inner automorphism.) This follows from the following one-cocycle property for a shift.

Lemma 4.4 *[19] Let* $B = \bigotimes_{\mathbf{Z}} M_n$ *and* σ *be the shift automorphism of* B *and define an action* γ *of* $\mathbf{T}^{n-1}$ *on* B *by* $\gamma_z = \bigotimes_{\mathbf{Z}} \mathrm{Ad}(1 \oplus z_1 \oplus z_2 \oplus \cdots \oplus z_{n-1})$ *for* $z = (z_1, \ldots, z_{n-1})$*, which commutes with* σ*. Let* $A = B^\gamma$ *be the fixed point algebra under* γ*. Then* $\sigma|A$ *has the one-cocycle property, i.e., for any unitary* $u = (u_n) \in B' \cap A^\infty$ *with* $\phi_i(u) = \lim_n \phi_i(u_n) = 1$ *there is a* $v \in \mathcal{U}(B' \cap A^\infty)$ *such that* $u = v\sigma(v^*)$*, where* $\phi_1, \ldots, \phi_n$ *are the characters of* A*.*

Any quasi-free flows with the Rohlin property are cocycle-conjugate with each other. The following is a generalization:

Proposition 4.5 *Let* α *and* β *be Rohlin flows on* $\mathcal{O}_n$*. If* $\mathcal{D}(\delta_\alpha^2)$ *and* $\mathcal{D}(\delta_\beta^2)$ *contain* C_n*, then they are cocycle-conjugate.*

We prove this as follows: We first show that there is an $h \in \mathcal{O}_n$ such that $h = h^*$ and $\delta_\alpha|C_n = \mathrm{ad}\, ih$. Thus we may just assume that $\delta_\alpha|C_n = 0 = \delta_\beta|C_n$, which means that α and β are not far from quasi-free or can be described in a rather explicit way. We use this fact to prove the claim.

Definition 4.6 *Let* A *be a* C^**-algebra and* α *a flow on* A*. Then* α_t *is said to be* α-invariantly approximately inner *if there is a sequence* (u_n) *in* $\mathcal{U}(A)$ *such that* $\alpha_t = \lim \mathrm{Ad}\, u_n$ *and* $\|\alpha_s(u_n) - u_n\|$ *converges to zero uniformly in* s *on every compact subset.*

For a free ultrfilter ω on $\mathbf{N}$, we define $c_\omega(A)$ to be the ideal of $\ell^\infty(A)$ consisting of $x = (x_n)$ satisfying $\lim_{n\to\omega} \|x_n\| = 0$ and set $A^\omega = \ell^\infty(A)/c_\omega(A)$. In the following theorem we use the classification theory of unital separable nuclear purely infinite simple C^*-algebras satisfying UCT and the fact that $A^\omega \cap A'$ is purely infinite, due to Kirchberg and Phillips [6, 7].

Theorem 4.7 *[16, 17] Let* A *be a unital separable nuclear purely infinite simple* C^**-algebra satisfying UCT and let* α *be a flow on* A*. Then the following conditions are equivalent.*

1. α *has the Rohlin property.*

2. $(A' \cap A^\omega_\alpha)^\alpha$ *is purely infinite and simple,* $K_0((A' \cap A^\omega_\alpha)^\alpha) \cong K_0(A' \cap A^\omega)$ *induced by the embedding, and* $\mathrm{Sp}(\alpha|A' \cap A^\omega_\alpha) = \mathbf{R}$*.*

3. *The crossed product* $A \times_\alpha \mathbf{R}$ *is purely infinite and simple and the dual action* $\hat{\alpha}$ *has the Rohlin property.*

4. *The crossed product* $A \times_\alpha \mathbf{R}$ *is purely infinite and simple and each* α_t *is* α*-invariantly approximately inner.*

If the above conditions are satisfied, it also follows that $K_1((A' \cap A^\omega_\alpha)^\alpha) \cong K_1(A' \cap A^\omega)$*, which is induced by the embedding.*

In the course of the proof we can prove the following, which might be interesting on its own. (We do not know about the K_1 version of it.)

Proposition 4.8 *Let $A, A' \cap A^\omega$ as above. Let e_0, e_1 be projections in $A' \cap A^\omega$ and let $(e_{\sigma,n})$ be a sequence of projections in A representing e_σ. Then e_0 and e_1 are equivalent in $A' \cap A^\omega$ iff for any finite subsets $\mathcal{P} \subset \mathcal{P}(A)$ and $\mathcal{U} \subset \mathcal{U}(A)$ there is an $\Omega \in \omega$ such that for any $n \in \Omega$, it follows that $[e_{\sigma,n}, p] \approx 0$, $[e_\sigma, u] \approx 0$, and*

$$[e_{0,n}p]_0 = [e_{1,n}p]_0, \quad [e_{0,n}u]_1 = [e_{1,n}u]_1$$

for all $p \in \mathcal{P}$ and $u \in \mathcal{U}$.

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