$X_A = \{ \ldots 1222113434343435666756 \ldots \}$

Classify such systems up to FLOW EQ?

- Suspension flow $S X_A = \{ (t,x) \in \mathbb{R} \times X_A \}/ \sim$ 

- Symbol expansion

... 1222 1134 0 340 340 403 403 5666756

- For IRREDUCIBLE $A$ of type I

$X_A \cong X_B \iff \begin{cases} \text{col}(I - A^t) = \text{col}(I - B^t) \\ \det(I - A^t) = \det(I - B^t) \end{cases}$

$O_A \otimes K \cong O_B \otimes K \implies \text{col}(I - A^t) \cong \text{col}(I - B^t)$

- Reducibility structure \sim ideals in $A$ in $O_A$
PROMISING STRATEGY

$O_A$, for $A$ type II falls into class studied by Kirchberg and Rørdam;

$O_A \otimes O_{\infty} = O_A$

Can we apply

THEOREM Kirchberg

Let $A$ and $B$ be stable, separable & nuclear

$A \otimes O_{\infty} = B \otimes O_{\infty}$\quad \text{if } \quad \text{Prim } A \cong X \cong \text{Prim } B

\quad \exists \varphi \in \text{KK}(X; A, B)

We just need a UCT

$\varphi : \text{KK}(X; A, B) \rightarrow \text{Hom}(K^*(A), K^*(B))$
INPUT FROM DYNAMICS

Boyle-Huang Isomorphism theorem for reducible systems

Restorff Classification theorem for non-simple $\mathfrak{A}$, A type $(I)$

The invariant:

\[ K_1(\text{circle}) \to K_0(\square) \to K_0(\text{circle}) \to K_0(\text{circle}) \]

\[ K_0(\text{circle}) \to K_0(\square) \]
LIFTING AUTOMORPHISMS

Theorem Rørdam

Essential extensions $X$ of purely injective, simple, separable and nuclear C*-algebras $A$, $B$

$0 \rightarrow B \rightarrow X \rightarrow A \rightarrow 0$

are classified up to stable isomorphisms by

$K_0(B) \rightarrow K_0(X) \rightarrow K_0(A)$
$\downarrow$

$K_1(A) \leftrightarrow K_1(X) \leftrightarrow K_1(B)$

Question

Do isomorphisms lift?

Answer

Sure! [E-Restorff]
CAVEAT Dadarlot-E
There exists a C′-algebra C, C ⊗ C
and \( f \in \text{Aut}(K_0(C) \oplus K_1(C)) \) which
does not lift even though

\[-f(K_0(\mathbb{Z}) \oplus K_1(\mathbb{Z})) = K_0(\mathbb{Z}) \oplus K_1(\mathbb{Z}) \quad \forall \mathbb{Z} \in \mathbb{Z} \]

\[-K_0(\mathbb{Z}) \subset K_0(C), K_1(\mathbb{Z}) \subset K_1(C) \quad \forall \mathbb{Z} \in \mathbb{Z} \]

OBSERVATION
Knowing that automorphisms lift
leads easily to unital version of
Rordam's theorem.
\[ \text{A UCT} \]


\[ X_2 = \{ \bullet \} \text{ with } \mathcal{A}, \mathcal{B} \in \mathcal{N} \]

\[ 0 \rightarrow \text{Ext}(\emptyset, \emptyset, \emptyset) \]

\[ \rightarrow \text{KU}(X_2; \mathcal{A}, \mathcal{B}) \]

\[ \rightarrow \text{Hom}(\emptyset, \emptyset, \emptyset) \]

\[ \rightarrow 0 \]
Bonkat's Method

- Category of 6-periodic complexes is abelian
- Projectives: exact complexes of free groups
- Enough projectives
- Finite projective dimension: exact
- Geometric resolutions
TO DO

- $\Lambda(A) = K_0(A) \oplus K_1(A) \oplus \bigoplus_{i,m} K_i(A)$

$K(A) \Rightarrow K(B)$

-Guess
For $X$ considered by Random

$0 \rightarrow \text{Imm}(X) \rightarrow \text{Aut}(X) \rightarrow$

$\text{Aut}(\text{K}(B) \Rightarrow \text{K}(X) \Rightarrow \text{K}(A)) \rightarrow 0$

$K_{sp}(A) \Leftarrow K_{sf}(X) \Rightarrow K_{sf}(B)$

-Guess
For $A, B$ considered by Booklet, with finitely generated $\mathbb{N}$-groups

$\text{KU}(X; A, B) \cong \text{Hom}(\text{KU}, \text{KU})$
Not a conjecture

\[ 0 \to \text{Ext}(K_{f, \#}(A), K_{f, \#}(B)) \to \text{KK}(X; A, B) \]

\[ \to \text{Hom}(K_{f, \#}(A), K_{f, \#}(B)) \to 0 \]