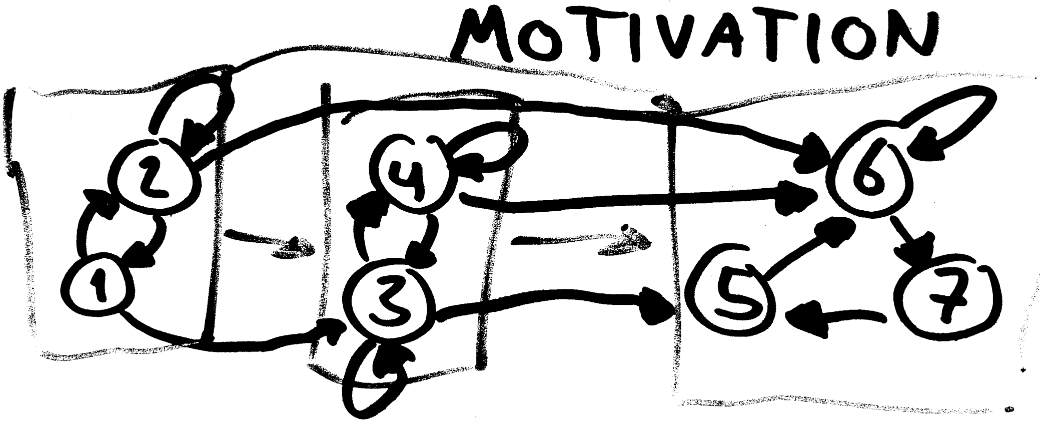
MOTIVATION

A: 

$$\begin{bmatrix} 0&1&1&0&0&0&0\\ 1&1&0&0&0&1&0\\ 0&0&1&1&1&0&0\\ 0&0&1&1&0&1&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&1&1&0\\ 0&0&0&0&0&0&1 \end{bmatrix}$$

$X_A = \{\cdots 1222113434344343566675 6\cdots\}$

Classify such systems up to FLOW EQ?

- Suspension flow $SX_A = \{(t,x)\in\mathbb{R}\times X_A\}/(t,x)\sim(t+1,\sigma(x))$

- Symbol expansion

  $\cdots 1222113 4\underline{0}34\underline{0}34\underline{0}4\underline{0}34\underline{0}35666756$

- For IRREDUCIBLE A of type I

  $$X_A \underset{FE}{\simeq} X_B \Leftrightarrow \begin{cases} \operatorname{cok}(I-A^t) \cong \operatorname{cok}(I-B^t) \\ \det(I-A^t) = \det(I-B^t) \end{cases}$$

  $$\mathcal{O}_A \otimes \mathbb{K} \cong \mathcal{O}_B \otimes \mathbb{K} \Leftrightarrow \operatorname{cok}(I-A^t) \cong \operatorname{cok}(I-B^t)$$

- Reducibility structure in A ~ Ideals in $\mathcal{O}_A$

PROMISING STRATEGY

$\mathcal{O}_A$, for A type II falls into class studied by Kirchberg and Rørdam;

$$\mathcal{O}_A \otimes \mathcal{O}_\infty \cong \mathcal{O}_A$$

<u>Can we apply</u>-

THEOREM Kirchberg

Let $\mathfrak{A}$ and $\mathfrak{B}$ be stable, separable & nuclear

$$\mathfrak{A}\otimes\mathcal{O}_\infty \cong \mathfrak{B}\otimes\mathcal{O}_\infty \iff \begin{cases} \operatorname{Prim}\mathfrak{A} \cong X \cong \operatorname{Prim}\mathfrak{B} \\ \exists\, \varphi \in KK(X;\mathfrak{A},\mathfrak{B})^{-1} \end{cases}$$

<u>We just need a UCT</u>

$$\ker^{\cdot} \hookrightarrow KK(X;\mathfrak{A},\mathfrak{B}) \twoheadrightarrow \mathrm{Hom}(K^{\cdot}(\mathfrak{A}), K^{\cdot}(\mathfrak{B}))$$

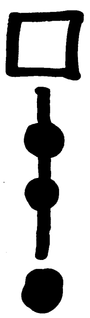
INPUT FROM DYNAMICS

Boyle-Huang Isomorphism theorem for reducible systems

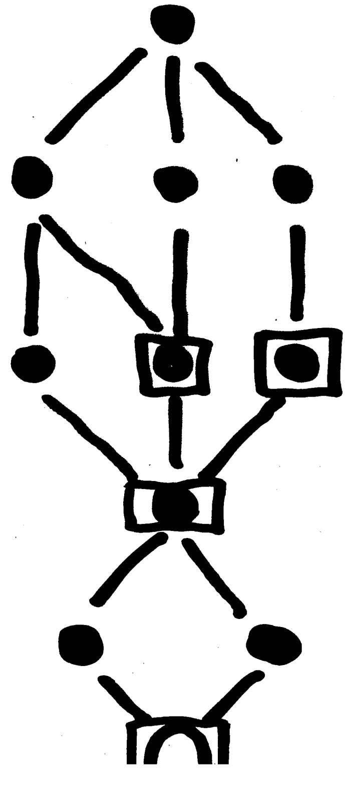
Restorff Classification theorem for non-simple $\mathcal{O}_A$, A type (II)

The invariant:

$$K_1(\bullet/\square) \to K_0(\square) \to K_0(\bullet) \to K_0(\bullet/\square)$$



$$K_0(\bullet) \to K_0(\square)$$



LIFTING AUTOMORPHISMS

THEOREM Rørdam

Essential extensions $\underline{\mathcal{X}}$ of purely infinite, simple, separable and nuclear C^*-algebras $\mathcal{A}$, B

$$0 \to B \to \mathcal{X} \to \mathcal{A} \to 0$$

are classified up to stable isomorphisms by

$$\begin{array}{ccccc} K_0(B) & \to & K_0(\mathcal{X}) & \to & K_0(\mathcal{A}) \\ \uparrow & & & & \downarrow \\ K_1(\mathcal{A}) & \leftarrow & K_1(\mathcal{X}) & \leftarrow & K_1(B) \end{array}$$

QUESTION

Do isomorphisms lift?

ANSWER

Sure! [E-Restorff]

COMMENTS

CAVEAT Dadarlat-E

There exists a C^*-algebra $\mathcal{E}$, $\mathcal{E}\otimes\mathcal{O}_\infty\cong\mathcal{E}$ and $\underline{f}\in \mathrm{Aut}(K_0(\mathcal{E})\oplus K_1(\mathcal{E}))$ which does not lift even though

- $f(K_0(\mathcal{J})\oplus K_1(\mathcal{J}))=K_0(\mathcal{J})\oplus K_1(\mathcal{J})$ $\forall\, \mathcal{J}\triangleleft\mathcal{E}$
- $K_0(\mathcal{J})\hookrightarrow K_0(\mathcal{E})$, $K_1(\mathcal{J})\hookrightarrow K_1(\mathcal{E})$ $\forall\, \mathcal{J}\triangleleft\mathcal{E}$

OBSERVATION

Knowing that automorphisms lift leads easily to unital version of Rørdam's theorem.

A UCT

Bonkat (PhD thesis, Münster 2002)

$X_2 = \left\{ \begin{array}{c} \bullet \\ | \\ \bullet \\ | \\ \circ \end{array} \right\}$. With $\mathcal{A}, \mathcal{B} < \mathcal{N}$

$$0 \to \mathrm{Ext}\left(\begin{tikzcd} \bullet \arrow[r] & \bullet \arrow[r] & \bullet \arrow[d] \\ \circ \arrow[u] & \circ \arrow[l] \arrow[u, phantom, "\mathcal{A}"] & \circ \arrow[l] \end{tikzcd}, \begin{tikzcd} \circ \arrow[r] & \circ \arrow[r] & \circ \arrow[d] \\ \bullet \arrow[u] & \bullet \arrow[l] \arrow[u, phantom, "\mathcal{B}"] & \bullet \arrow[l] \end{tikzcd} \right)$$

$$\to KK(X_2; \mathcal{A}, \mathcal{B})$$

$$\to \mathrm{Hom}\left(\begin{tikzcd} \bullet \arrow[r] & \bullet \arrow[r] & \bullet \arrow[d] \\ \circ \arrow[u] & \circ \arrow[l] \arrow[u, phantom, "\mathcal{A}"] & \circ \arrow[l] \end{tikzcd}, \begin{tikzcd} \bullet \arrow[r] & \bullet \arrow[r] & \bullet \arrow[d] \\ \circ \arrow[u] & \circ \arrow[l] \arrow[u, phantom, "\mathcal{B}"] & \circ \arrow[l] \end{tikzcd} \right)$$

$$\to 0$$

BONKAT'S METHOD

- Category of 6-periodic complexes is abelian
- Projectives: exact complexes of free groups
- Enough projectives
- Finite projective dimension: exact
- Geometric resolutions

$$[\qquad]^{-1}$$

$$\Rightarrow [\qquad]^{-1}$$

$$\Rightarrow 0$$

$+$

IN $\mathcal{E}$, ONE MAY CHOOSE $\Phi : A \otimes \mathcal{O}_n \to B \otimes \mathcal{O}_n$ with $[\Phi] = a$

TO DO

- $\underline{K}(\mathcal{A}) = K_0(\mathcal{A}) \oplus K_1(\mathcal{A}) \oplus \bigoplus_{i,n} K_i(\mathcal{A}; \mathbb{Z}_n)$

$\underline{K}(\mathcal{A}) \to \underline{K}(\mathcal{B})$

- GUESS

For X considered by Rørdam

$$0 \to \overline{\mathrm{Inn}(X)} \to \mathrm{Aut}(X) \to \mathrm{Aut}\left(\begin{tikzcd} \underline{K}(\mathcal{B}) \arrow[r] & \underline{K}(X) \arrow[r] & \underline{K}(\mathcal{A}) \arrow[d] \\ \underline{K}_{+1}(\mathcal{A}) \arrow[u] & \underline{K}_{+1}(X) \arrow[l] & \underline{K}_{+1}(\mathcal{B}) \arrow[l] \end{tikzcd} \right) \to 0$$

- GUESS

For $\mathcal{A}, \mathcal{B}$ considered by Bonkat, with finitely generated K-groups

$$KK(X_2; \mathcal{A}, \mathcal{B}) \cong \mathrm{Hom}($$

NOT A CONJECTURE

$$0 \to \mathrm{Ext}\left(K_{\mathrm{filH}_X}(A), K^{*1}_{\mathrm{filH}_X}(B)\right)$$

$$\to KK(X; A, B)$$

$$\to \mathrm{Hom}\left(K_{\mathrm{filH}_X}(A), K_{\mathrm{filH}_X}(B)\right) \to 0$$