THE ABEL SYMPOSIUM 2019

ÅLESUND, SCANDIC PARKEN

The scientific and organizing committee:

José Figueroa-O'Farrill (Edinburgh) ∘ Sigbjørn Hervik (Stavanger)
Boris Kruglikov (Tromsø) ∘ Irina Markina (Bergen) ∘ Jan Slovák (Brno)
Dennis The (Tromsø) ∘ Bent Ørsted (Aarhus)

	Mon 24/6	Tue 25/6	Wed 26/6	Thu 27/6	Fri 28/6
08:00	Breakfast				
09:00	Agricola	Nurowski	Excursion	Grong	Munthe-Kaas
10:00	Alekseevsky	Eastwood		Hwang	Singer
11:00	Calderbank	Ferapontov		Juhl	Santi
12:00	Lunch			Lunch	
14:00	Čap	Fino		Lauret	Schlichenmaier
15:00	Cowling	Garcia-Rio		Leistner	Serganova
15:45	Coffee			Coffee	
16:15	Doubrov	Gilkey		Lychagin	Neusser
17:15	Melnick	Gover		Dunajski	Wylleman
19:00	Dinner				

Remarks:
1. Opening will happen on Monday at 08:50. Ask reception for directions.
2. The time and space for lunch and dinner to be confirmed.
3. Dinner on Wednesday will start after the excursion and there will be a banquet.
∞. Further information will be provided at the opening.

More info on web-page: abelsymposium.no.

Titles & Abstracts:

Ilka Agricola (University of Marburg)

Title: Generalizations of 3-Sasakian manifolds and skew torsion

Abstract: We define and investigate new classes of almost 3-contact metric manifolds, with two guiding ideas in mind: first, what geometric objects are best suited for capturing the key properties of almost 3-contact metric manifolds, and second, the newly defined classes should admit 'good' metric connections with skew torsion with interesting applications: these include a well-behaved metric cone, the existence of a generalized Killing spinor, and remarkable curvature properties.

Dmitri Alekseevsky (Institute for Information Transmission Problems of the Russian Academy of Sciences)

Title: Homogeneous sub-Riemannian manifolds

Abstract: For a homogeneous sub-Riemannian manifold $(M = G/H, D, g)$ we study two types of geodesics: H-geodesics, defined as projection to M of the integral curves of the sub-Riemannian Hamiltonian and S-geodesics, which are defined as the geodesics of the partial Shouten connection and describes the evolution of a classical mechanical system with non-holonomic constraints. In particular, we study homogeneous sub-Riemannian geodesics. A sufficient condition that H-geodesics coincides with S-geodesics is given. Some results on classification of invariant bracket generated sub-Riemannian and sub-Lorentzian structures on homogeneous manifolds $M = G/H$ of a semisimple Lie group G will be presented.

David Calderbank (University of Bath)

Title: GL_2 geometry and quaternionic manifolds with symmetry

Abstract: Moduli spaces of submanifolds of a complex manifold typically carry special differential geometric structures. In the case that the submanifolds are projective lines, the corresponding geometric structures are integrable GL_2 geometries. Prototypical examples include Einstein–Weyl geometry and quaternionic geometry. This talk will introduce a general setting for integrable G-geometry, and explore how integrable GL_2 geometries in different dimensions are related by symmetry reduction and generalizations thereof. In particular, we see that all (complex) integrable GL_2 geometries arise (locally) as symmetry reductions of (complexified) quaternionic geometry.

Andreas Čap (University of Vienna)

Title: The bundle of Weyl structures

Abstract: This talk reports on joint work in progress with Thomas Mettler (Frankfurt). We study the bundle of Weyl structures associated to a parabolic geometry that was originally constructed by M. Herzlich in order to study distinguished curves. Via an improved construction of the canonical connection available in this setting, we obtain a simple geometric description of all parts of the theory of Weyl structures.

The second part of the talk will be devoted to the analysis of the intrinsic geometric structure of the bundle of Weyl structures for parabolic geometries associated to |1|-gradings (AHS-structures). In particular, it carries a natural almost bi-Lagrangean structure, which combines an almost symplectic structure and a pseudo-Riemannian metric of split signature that both are parallel for the canonical connection. Using the calculus developed in the first part, we show that a symplectic structure is obtained iff the initial AHS structure is torsion-free and that in this case one obtains an Einstein metric. In the end of the talk, I'll outline how this relates to fully non-linear invariant PDE that are intrinsic to the initial AHS structure.

Michael Cowling (University of New South Wales)

Title: Lie groups as metric spaces

Abstract: Lie groups and Riemannian geometry have been linked for a long time. More recently, however, Lie groups equipped with non-Riemannian distances have appeared in various contexts. This talk aims to survey some of what is known about left-invariant distance functions on connected Lie groups, and present some recent results. Much of this work is joint with Ville Kivioja, Enrico Le Donne, Sebastiano Nicolussi Golo, and Alessandro Ottazzi.

Boris Doubrov (Belarusian State University)

Title: Extrinsic geometries and linear PDEs on filtered manifolds

Abstract: We develop a general theory of submanifolds in homogeneous spaces equipped with a filtration of a tangent bundle. Similar to he intrinsic geometry of structures on filtered manifolds, this includes the construction of a canonical frame and the description of the space of invariants. In case, this filtered homogeneous space is embedded to a flag variety (preserving the filtration), we construct the natural system of linear PDEs associated with this embedding and show that this gives an equivalence of two categories. Finally, the generalizations to the infinite-dimensional case are discussed.

Maciej Dunajski (University of Cambridge)

Title: Conformally isometric embeddings

Abstract: I shall discuss some necessary and sufficient conditions for existence of a locally isometric embedding of a Ricci–flat Riemannian four-manifolds in conformally-flat five-manifolds. There is some interesting maths - the obstructions arise from a prolongation connection - as well as physics: the Hawking temperature of the Schwarzschild metric agrees with the Unruh temperature measured by an observer moving along conformal circles in five dimensions. This is based on a joint work with Paul Tod.

Michael Eastwood (University of Adelaide)

Title: The parabolic geometry of a flying saucer

Abstract: The motion of a flying saucer is restricted by the three-dimensional geometry of the space in which it moves. In this way, various parabolic geometries and Lie algebras emerge from thin air. I shall discuss the geometry of the particular thin air needed so that Engel's 1893 construction of the exceptional Lie algebra G_2 emerges. This is joint work with Pawel Nurowski.

Evgeny Ferapontov (Loughborough University)

Title: On integrability of dispersionless Hirota type equations

Abstract: I will review two recent results on integrability of dispersionless Hirota type equations:

1. Generic integrable Hirota type equation in 3D coincides with the equation of the genus 3 hyperelliptic divisor.
2. Every integrable Hirota type equation in 4D is necessarily of Monge-Ampere type, and reduces to one of the 6 known canonical forms of heavenly type equations governing self-dual Ricci flat metrics.

References:

1. E.V. Ferapontov, L. Hadjikos and K.R. Khusnutdinova, *Integrable equations of the dispersionless Hirota type and hypersurfaces in the Lagrangian Grassmannian*, International Mathematics Research Notices, 496-535 (2010).
2. B. Doubrov and E.V. Ferapontov, *On the integrability of symplectic Monge-Ampere equations*, Journal of Geometry and Physics **60**, 1604-1616 (2010).
3. F. Clery, E.V. Ferapontov, *Dispersionless Hirota equations and the genus 3 hyperelliptic divisor*, arXiv: 1804.07724.
4. E.V. Ferapontov, B. Kruglikov, V.S. Novikov, *Integrability of dispersionless Hirota type equations in 4D and the symplectic Monge-Ampere property*, arXiv:1707.08070.

Anna Fino (Università di Torino)

Title: Closed G_2-structures and Laplacian flow

Abstract: I will review known examples of compact 7-manifolds admitting a closed G_2-structure. Moreover, I will discuss some results on the behaviour of the Laplacian G_2-flow starting from a closed G_2-structure whose induced metric satisfies suitable extra conditions.

Eduardo Garcia-Rio (University of Santiago de Compostela)

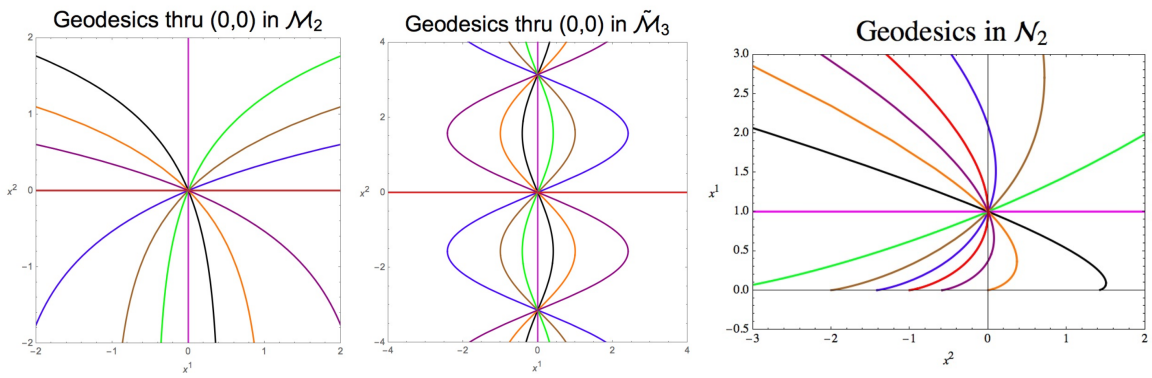
Title: Conformally Einstein metrics in dimension four

Abstract: A Riemannian metric g is said to be conformally Einstein if there is an Einstein representative in the conformal class $[g]$. The necessary and sufficient conditions for a metric to be conformally Einstein, obtained by Brinkmann at the beginning of the twentieth century, are summarized as an overdetermined system of PDEs. Besides its apparent simplicity their integration is surprisingly difficult and many open questions are still open. The purpose of this talk is to give a complete description of conformally Einstein homogeneous metrics in dimension four, thus generalizing previous work of Jensen on the classification of homogeneous Einstein four-manifolds.

Peter Gilkey (University of Oregon)

Title: Geodesic completeness of homogeneous affine surfaces

Abstract: If γ is a curve in an affine surface $\mathcal{M} = (M, \nabla)$, then the geodesic equation is $\ddot{\gamma}^i + \Gamma_{jk}{}^i \dot{\gamma}^j \dot{\gamma}^k = 0$ for all i. There are 3 families of homogeneous affine surfaces. The Type $\mathcal{A}$ are the left invariant structures on the translation group $\mathbb{R}^2$, the Type $\mathcal{B}$ are the left invariant structures on the $ax + b$ group, and the Type $\mathcal{C}$ arise as the underlying affine structure of a surface of constant sectional curvature. We discuss complete classification results for the geodesically complete Type $\mathcal{A}$ and partial results for the Type $\mathcal{B}$ surfaces. This is work in progess with Daniela D'Ascanio and Pablo Pisani (Dept. Fisica Universidad Nacional de la Plata, Argentina) and X. Valle-Regueiro (Universidade de Santiago, Espana).



Rod Gover (University of Auckland)

Title: Distinguished curves and integrability in Riemannian, conformal, and projective geometry

Abstract: A new characterisation is described for the unparametrised geodesics, or distinguished curves, for affine, pseudo-Riemannian, conformal, and projective geometry. The characterisation is a type of moving incidence relation and most importantly it leads naturally to a very general theory and construction of quantities that are necessarily conserved along the curves. In this the usual role of Killing tensors and conformal Killing tensors is recovered, but the construction shows that a significantly larger class of equation solutions can also yield curve first integrals. In particular any normal solution to an equation from the class of first BGG equations can potentially yield such a conserved quantity. For some equations the condition of normality is not required. For nowhere-null curves in pseudo-Riemannian and conformal geometry additional results are available and these are via a fundamental tractor-valued invariant of such curves. This quantity is parallel if and only if the curve is an unparametrised conformal circle.

This talk is based on joint work with Daniel Snell and Arman Taghavi-Chabert.

Erlend Grong (University of Bergen)

Title: Curvature for sub-Riemannian manifolds

Abstract: Sub-Riemannian geometry considers spaces where only curves tangent to a given subbundle have finite length. This geometry is related to second order partial differential operators that are hypoelliptic but not elliptic.

There has been several recent breakthroughs in the effort defining curvature in this geometry. In our talk, we will go through several of the most recent developments. The talk will focus on results relating to comparison theorems and connections compatible with the sub-Riemannian structure.

Jun-Muk Hwang (Korea Institute for Advanced Study)

Title: Recognizing symplectic Grassmannians by their varieties of minimal rational tangents

Abstract: Let G/P be a rational homogeneous space of Picard number 1 and let X be a Fano manifold of Picard number 1. The question we are interested in is the following: if the variety of minimal rational tangents at a general point of X is isomorphic to that of G/P, is X biholomorphic to G/P?

Ngaiming Mok gave an affirmative answer when G/P is a Hermitian symmetric space or a homogeneous contact manifold. Mok's argument was generalized by Hong-Hwang to any G/P associated with a long root. In these cases, the underlying geometric structure modeled on G/P is a parabolic geometry, for which local differential geometric machinery is well-developed in the classical works of Tanaka. The question has been open for the other G/P, i.e. symplectic Grassmannians or two cases of F_4-homogeneous spaces. The main difficulty in the remaining cases is that the underlying geometric structure is no longer a parabolic geometry and certain degenerate structures may occur in a neighborhood of a minimal rational curve. Because of the potential degeneracy, standard Tanaka theory is not available. In a collaboration with Qifeng Li, we overcome this difficulty by constructing a Cartan connection associated to the geometric structure in a neighborhood of a minimal rational curve, under the assumption that certain vector bundles arising from Spencer complexes do not have nonzero sections. Using this construction, we settle the case of symplectic Grassmannians. Our argument also works for odd-symplectic Grassmannians.

Andreas Juhl (Humboldt-Universität Berlin)

Title: On the structure of Branson–Gover and Q-curvature operators on differential forms

Abstract: In 2004, T. Branson and R. Gover introduced generalizations to operators acting on differential forms of the famous conformal powers of the Laplacian (GJMS operators) and of the related scalar curvature invariant known as Branson's Q-curvature. Their definitions rest on the conformal tractor calculus. We found that these constructions admit an alternative description in terms of appropriate generalizations of the notion of residue families. In the scalar case, this notion has been much introduced by the author and studied much in recent years. In the form case, residue families are conformally covariant one-parameter families of differential operators which are curved versions of so-called symmetry breaking operators (a concept recently coined by T. Kobayashi). In the lecture, we describe these concepts, their interactions and some resulting new formulas for the critical Q-curvature operators.

Jorge Lauret (Universidad Nacional de Córdoba)

Title: Soliton geometric structures on Lie groups

Abstract: The concept of soliton, in its more general version, allows to find canonical or distinguished elements on a space of structures provided with an equivalence relation and an "optimal" tangent direction at each point. The use of the term "soliton" was initiated by R. Hamilton in the 80s to refer to self-similar solutions of the Ricci flow, but it is nowadays used in the study of geometric flows in many other areas of differential geometry, including symplectic, hermitian and special holonomy geometries. In this talk, we will survey on the role played by solitons in providing canonical geometric structures on Lie groups.

Thomas Leistner (University of Adelaide)

Title: Semi-Riemannian cones and their holonomy

Abstract: Cone constructions can be used to study certain overdetermined PDE on manifolds. An example is the classification of Riemannian manifolds with Killing spinors, which is based on three crucial results: (a) the observation that Killing spinors correspond to parallel spinors on the cone, (b) Gallot's result that a Riemannian cone over a complete manifold is flat if it decomposes as a Riemannian product, and (c) that Riemannian holonomy groups are completely reducible to the irreducible groups on Berger's list (for which the corresponding geometric structures are known). Generalising this strategy to manifolds with indefinite metrics faces the main difficulties that (b) does not generalise directly and that holonomy groups of indefinite metrics are not completely reducible as in (c). I will first review results about semi-Riemannian cones that do decompose into a semi-Riemannian product and give a generalisation of Gallot's theorem under some extra assumption. Then I will present more recent results about cones that do not decompose as a product. These either have irreducible holonomy, in which case a holonomy classification follows from Berger's list, or admit a parallel vector distribution on which the metric degenerates. We will show what this implies for the geometry of the base manifold and for the case of timelike cones over Lorentzian manifolds describe the possible holonomy groups.

Valentin Lychagin (UiT The Arctic University of Norway)

Title: On equivalence of linear differential operators

Abstract: We'll give conditions under which linear differential operators, acting in a line bundle over compact manifold, are globally equivalent with respect to automorphism group.

Karin Melnick (University of Maryland)

Title: A D'Ambra Theorem in conformal Lorentzian geometry

Abstract: D'Ambra proved in 1988 that the isometry group of a compact, simply connected, real-analytic Lorentzian manifold must be compact. I will discuss the recent theorem that the conformal group of such a manifold must also be compact, and how it relates to the Lorentzian Lichnerowicz Conjecture.

Hans Munthe-Kaas (University of Bergen)

Title: A survey of Lie–Butcher theory

Abstract: Lie–Butcher (LB) series is a unification of classical Lie series with B-series. B-series, named after J. Butcher [1] describe formal flows on Euclidean spaces, and is the main tool for studying algebra and geometry of numerical integration. Lie–Butcher series define formal flows on Lie groups and homogeneous spaces. The theory originated with the study of numerical integration on manifolds [4, 5], and has in recent years evolved into a rich mathematical theory combining algebraic structures with geometry of fibre bundles and applicatoins [8, 6, 2, 7]. We will in this talk survey this field, both from the theoretical side and also indicate applications to integration on manifolds, controlled differential equations and rough paths theory.

We will view LB theory from the perspectives of algebra, geometry and applications. Invariant connections on anchored bundles gives rise to three special algebras: preLie, postLie and postLie triple algebras, which on the geometric side correspond to Euclidean spaces, homogeneous spaces and symmetric spaces. The notion of a pre-Lie algebra originates from work of Vinberg [11] and Gerstenhaber [3], while post-Lie algebras appeared first in an algebraic context Vallette [10]. The free algebras are represented in terms of rooted trees. On the geometric side, we have recently characterised anchored fibre bundles admitting such algebras [7]. In particular we show that postLie algebras are intimately associated with action algebroids, generalising classical results of Nomizu [9].

References

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Katharina Neusser (Masaryk University)

Title: Projective geometry of Sasaki–Einstein structures and their compactification

Abstract: This talk will be concerned with Sasaki manifolds, which can be characterized as (pseudo-) Riemannian manifolds whose metric cone is Kähler. We will show that Sasaki manifolds admit a natural description in terms of projective differential geometry. In particular, we will see that Sasaki–Einstein manifolds may be characterized as projective manifolds equipped with certain unitary holonomy reductions of their canonical Cartan connections. This characterization will moreover allow us to describe a natural geometric compactification of complete non-compact (indefinite) Sasaki–Einstein manifolds. This talk is based on joint work with Rod Gover and Travis Willse.

Pawel Nurowski (Centrum Fizyki Teoretycznej PAN)

Title: The parabolic geometry of a car

Abstract: I will discuss the geometry of the configuration space of a car equipped with ideal tires. Such a physical system is nonholonomic, meaning that its velocities obey constraints that can not be integrated to the constraints on positions. These constraints are linear, which results in equipping the configuration space of a car with the structure of a bracket generating vector distribution. Surprisingly, two kinds of very natural movements of a car define an additional structure on this distribution, whose local symmetry is $\mathrm{SO}(2,3)$.

My talk will try to provide elementary arguments why this simple Lie group, which in particular is a conformal symmetry group of the 3-dimensional Minkowski space, appears as one of the features of any car.

Andrea Santi (University of Padova)

Title: Symmetry superalgebras in geometry and physics

Abstract: In the first part of the talk, I will report on the structure of Lie superalgebras generated by Killing spinors and discuss applications to the classification of supersymmetric backgrounds of $\mathrm{d} = 11$ supergravity. In particular, we will see that preserving more than half the supersymmetry implies the Einstein and Maxwell equations of $\mathrm{d} = 11$ supergravity. I will also elucidate the rôle played in this approach by a certain Spencer cohomology group, which defines the relevant notion of Killing spinor, and give the classification of $\mathrm{d} = 6$ maximally supersymmetric backgrounds. These are joint works with J. Figueroa-O'Farrill and P. de Medeiros.

In the second part of the talk, I will present the realization of the simple Lie superalgebra $\mathrm{G}(3)$ as supersymmetry of various geometric structures, most importantly super-versions of Hilbert–Cartan equation and Cartan's involutive PDE system that exhibit $\mathrm{G}(2)$ symmetry. I will briefly discuss non-holonomic superdistributions with growth vector $(2|4,1|2,2|0)$ obtained as super-deformations of rank 2 distributions in a 5-dimensional space, and show that the second Spencer cohomology group gives a binary quadric, thereby providing a "square-root" of Cartan's classical binary quartic invariant for $(2,3,5)$-distributions. This is a joint work with B. Kruglikov and D. The.

Martin Schlichenmaier (University of Luxembourg)

Title: N point Virsasoro algebras are multi-point Krichever-Novikov type algebras

Abstract: We show how the recently again discussed N-point Witt, Virasoro, and affine Lie algebras are genus zero examples of the multi-point versions of Krichever–Novikov type. These multi-point versions were introduced and studied by Schlichenmaier. Using this more general point of view, useful structural insights and an easier access to calculations can be obtained. The concept of almost-grading will yield information about triangular decompositions which are of importance in the theory of representations. The algebra of functions, vector fields, differential operators, current algebras, affine Lie algebras, Lie superalgebras and their central extensions show up. As special example the three-point case is given.

Vera Serganova (University of California, Berkeley)

Title: Homological tensor functors for representations of supergroups and superalgebras

Abstract: Let x be a self-commuting element in the Lie superalgebra $\mathfrak{g}$ and M be any representation of $\mathfrak{g}$. Then the cohomology of x in M defines a tensor functor to the category of representations over a smaller superalgebra, This functor has many applications: computation of dimension of irreducible representations, construction of abelian envelopes of Deligne's tensor categories, description of the ring of characters and classification of blocks in the category of representations of algebraic supergroups. Moreover, this functor can be upgraded to a functor from the category of representations of supergroups to the category of equivariant quasicoherent sheaves on a self-commuting cone of $\mathfrak{g}$ which leads to the theory of support varieties for supergroups. In this talk I will explain some of these applications.

Michael Singer (University College London)

Title: Boundary value problems for Einstein metrics

Abstract: Let M be a compact oriented d-dimensional manifold with boundary N. A natural geometric boundary value problem is to find an asymptotically hyperbolic Einstein metric g on (the interior of) M with prescribed 'conformal infinity' on N. A little more precisely, the problem is to find (Einstein) g with the boundary condition that $x^2 g$ tends to a metric h on N as x goes to 0, x being a boundary defining function for N. The prototype is the hyperbolic metric g on the ball, with conformal infinity the round metric on the boundary sphere. Since the pioneering work of Graham and Lee (1991) the problem has attracted attention from a number of authors.

In this talk, I shall explain a gauge-theoretic approach to the problem which works in dimension $d = 4$, and explain how it can be used to obtain some new results for this boundary value problem. Based on joint work with Joel Fine and Rafe Mazzeo.

Lode Wylleman (University of Stavanger)

Title: Invariantly-defined curvature properties and integrability in general relativity

Abstract: The search for metrics with prescribed invariantly-defined properties of the curvature tensor of the Levi-Civita connection is one of great importance in general relativity. A fruitful technique is to work with a rigid frame formalism (or a partially covariant version thereof) adapted to the properties under study. The Bianchi and relevant Ricci identities are written in terms of the directional derivatives (or associated partially covariant operators), the relevant connection coefficients and the Riemann tensor components.

For a general metric these form a set of equations equivalent to a closed linear PDE system. Invariantly-defined properties translate to constraints on this system. By reclosing the extended system (integrability analysis) one obtains an invariant description of the class of all metrics obeying the properties, and finally tries to find a suitable set of coordinates to represent them (integration). A more detailed outline of this technique will be presented and several examples provided.