The Abel Symposium 2015
Operator Algebras and Applications
7-11 August, 2015, Coastal Express, Norway
Titles and abstracts of talks

Erling Størmer (University of Oslo)

Uffe Haagerup - In Memoriam

Erik Christensen (University of Copenhagen, Denmark)

Reflexive algebras of weakly differentiable operators

Let D be an unbounded self-adjoint operator on a Hilbert space H. A bounded operator x in a subalgebra A of $B(H)$ is $n-$times weakly $D-$differentiable and belongs to the algebra $C^n(A, D)$ if for any pair of vectors ξ, η the function $t \to \langle e^{itD}xe^{-itD}\xi, \eta\rangle$ is $n-$times differentiable on $\mathbb{R}$. The concept of weak differentiability is different from the basic one, named uniform $D-$differentiability, and we give a characterization of the difference. We also show that for any von Neumann algebra $\mathcal{M}$ on H, there exists a representation of $C^n(\mathcal{M}, D)$ on $H \otimes \mathbb{C}^{n+1}$ such that the image is a reflexive subalgebra of $B(H \otimes \mathbb{C}^{n+1})$. We remind you that an algebra $\mathcal{R}$ of bounded operators is reflexive if it equals the algebra of all bonded operators which leave each space in the lattice of invariant subspaces for all the operators in $\mathcal{R}$ invariant. In particular if $\mathcal{R}$ is reflexive and self-adjoint, then $\mathcal{R}$ is a von Neumann algebra. To get a concrete understanding you should think of the circle and differentiation with respect to arc length.

Alain Connes (Collège de France & I.H.E.S, France, Ohio State University, USA)

Geometry and the Quantum

I will present in my talk the recent joint work with Chamseddine and Mukhanov, in which we introduce an equation on operators in Hilbert space whose solutions yield 4-dimensional manifolds, and where the spectral action gives gravity coupled with the standard model. The picture that emerges is that the Euclidean space-time unfolds to macroscopic size from the product of two 4-spheres of Planckian size as a butterfly unfolds from its chrysalis.

Joachim Cuntz (Westfälische Wilhelms-Universität Münster, Germany)

C^*-algebras arising from irreversible algebraic systems

We give a survey of work over the past years, on C^*-algebras associated with systems arising from algebraic number theory or ergodic theory.

Siegfried Echterhoff (Westfälische Wilhelms-Universität Münster, Germany)

K-theory for exotic crossed products

We report on joint work with Alcides Buss and Rufus Willett about functorial properties of exotic crossed-product functors with applications to the computation of K-theory groups of certain exotic group algebras and crossed products by K-amenable groups.

Søren Eilers (University of Copenhagen, Denmark)

The classification problem for Cuntz-Krieger algebras

The classification problem for Cuntz-Krieger algebras has a long and prominent history. Indeed, Rørdams classification in 1995 of the simple such C*-algebras by appealing to results in symbolic dynamics paved the way for the sweeping generalization by Kirchberg and Phillips to all simple, nuclear, separable and purely infinite C*-algebras, and Restorffs generalization in 2006 to the case of Cuntz-Krieger algebras with finitely many ideals (equivalently, of real rank zero) was a key inspiration for the recent surge in results concerning nonsimple purely infinite C*-algebras.

I will explain how recent progress on the understanding of this problem - arising from the ambition to generalize Restorff's result to all unital graph C*-algebras - leads to a complete classification of the entire class of Cuntz-Krieger algebras.

This is joint work with Restorff, Ruiz and Sørensen.

George A. Elliott (University of Toronto, Canada)

Simple, unital, separable, UCT C-algebras with finite decomposition rank are classifiable*

Following on their recent TA(point-line) algebra classification, Gong, Lin, and Niu together with me have shown that the class of the title coincides with their class. By work of Niu, Santiago, Tikuisis, and me (together with earlier results) it follows that this class also coincides with the class of separable simple unital ASH algebras with slow dimension growth.

Thierry Giordano (University of Ottawa, Canada)

$\mathbb{Z}^d$-Odometers and cohomology

$\mathbb{Z}$-Odometers are well-known examples of Cantor minimal systems. In particular, any two orbit equivalent $\mathbb{Z}$-odometers are conjugate. In this talk, we will use two equivalent formulations of $\mathbb{Z}^d$-odometers to show that there exist orbit equivalent $\mathbb{Z}^2$-odometers which are not conjugate.

This is joint work with I. Putnam and C. Skau.

Astrid an Huef (University of Otago, New Zealand)

Toeplitz algebras of Baumslag-Solitar semigroups

Jack Spielberg has recently shown that the Baumslag-Solitar groups are quasi-lattice ordered in the sense of Nica. Thus they have tractable Toeplitz algebras, and each of these algebras carries a natural action of the real line. I will talk about the equilibrium (KMS) states of these algebras. For inverse temperatures larger than a critical value, there is a large simplex of KMS states parametrised by probability measures on the unit circle. At the critical value, and under a mild hypothesis, there is a phase transition in which this simplex collapses to a singleton.

Vaughan Jones (Vanderbilt University, USA)

Fun with the Thompson group

We show how to associate a knot or link to every element of Thompson's group F and that every knot or link arises in this way. The links arise as coefficients of (necessarily infinite dimensional) representations of F and by changing the input data from a crossing to an element of a subfactor we obtain an intriguing family of unitary representations of F.

David Kerr (Texas A&M, USA)

Quasidiagonality, unique ergodicity, and crossed products

Recently Downarowicz, Huczek, and Zhang proved that every countable amenable group can be tiled by translates of finitely many Følner sets. I will explain how this result can be applied in conjunction with arguments of Ozawa, Rørdam, and Sato to show that every elementary amenable group admits many uniquely ergodic actions whose crossed product is quasidiagonal.

Marcelo Laca (University of Victoria, Canada)

Multiplicative action of algebraic units on ideals

For each number field, we study the multiplicative action of the group of units on the ring of algebraic integers and on ideals representing each ideal class. Using a recent theorem of Neshveyev, we show that the extremal KMS states of the C*-dynamical systems of Cuntz, Deninger and Laca are parametrized by pairs (μ, κ) consisting of an ergodic invariant measure μ on the dual of an ideal and a character κ of the associated μ-a.e. constant isotropy subgroup of units. This naturally leads to the consideration of groups of quasi-hyperbolic toral automorphisms, revealing that for most number fields the problem of deciding whether extremal KMS states arise exclusively from rational points and from Haar measure on a torus is a generalized version of Furstenberg's "$\times_2$ - $\times_3$" conjecture. Motivated by this, we discuss closed invariant sets and ergodic invariant probability measures for the action of units on ideals in terms of the class number and the signature of a number field.

This is joint work with J. Maria Warren (Victoria).

Xin Li (Queen Mary University of London, UK)

Continuous orbit equivalence rigidity

We discuss the notion of continuous orbit equivalence, explain its connection to operator algebras, and explore rigidity phenomena in this context.

Hiroki Matui (Chiba University, Japan)

Examples of topological full groups

I will begin with the definition of topological full groups and explain some concrete examples of them. Especially, I will discuss isomorphism theorem, homology groups of groupoids, connection to K-theory, abelianization, etc.

Ryszard Nest (University of Copenhagen, Denmark)

Perturbation isomorphisms and their analytic properties

As is well known, the index class in K_0^{top} of a Fredholm complex $\mathcal{C} = (H_i, d_i)$ of Hilbert spaces is invariant under finite rank (in fact compact) perturbations of the complex. A K-algebraic invariant associated to such a complex is the determinant line of its homology groups,

$$Det(\mathcal{C}) = \prod_k \Lambda^{top} H_{2k}(\mathcal{C}) \otimes \prod_k \Lambda^{top} H_{2k+1}(\mathcal{C})^*.$$

This determinant line has similar invariance properties to the index class. The perturbation isomorphism is a canonical isomorphism of the determinant lines associated to a finite rank perturbations of $\mathcal{C}$.

We will sketch the construction of the perturbation isomorphisms and study their analytic properties. In particular, we will use them to provide a holomorphic structure on the determinant lines of the homology spaces over the space of Fredholm complexes, generalising the known construction due to Quillen in the case of a single Fredholm operator. Similarly to the six term exact sequence in the topological K-theory, a morphism of Fredholm complexes leads to a torsion isomorphism of certain determinant lines which we show to be analytic with respect to the holomorphic structure described above.

This is joint work with Jens Kaad.

Ian Putnam (University of Victoria, Canada)

A homology theory for Smale spaces

David Ruelle defined Smale spaces to describe the dynamics of Smale's Axiom A systems on their basic sets. They include shifts of finite type, Anosov diffeomorphisms, various solenoids and the space of a substitution tiling such as the Penrose tilings. Manning showed that the Artin-Mazur zeta function for such systems is rational and this prompted Bowen to propose the existence of a homology theory for such systems which would provide a Lefschetz-type theorem. For shifts of finite type, Krieger showed how to construct a C*-algebra from the dynamics. This was an AF-algebra and its K-theory answered Bowen's question. Here we show how to extend Krieger's invariant to all (non-wandering) Smale spaces.

Iain Raeburn (University of Otago, New Zealand)

Equilibrium states on graph algebras

We consider operator-algebraic dynamical systems consisting of an action of the real numbers on a C*-algebra. The equilibrium states of the system are those that satisfy a commutation relation known as the KMS condition.

There has recently been considerable interest of the KMS states of systems based on Toeplitz algebras of various sorts. This analysis has been particularly effective for algebras associated to directed graphs and their generalisations. Here we discuss some these results for finite directed graphs. For the Cuntz-Krieger algebras of irreducible directed graphs, the KMS states were analysed by Enomoto, Fujii and Watatani in the 1980s. Systems based on Toeplitz algebras typically have a richer supply of KMS states. We will discuss this analysis in the context of the algebras of finite graphs, with an emphasis on what happens for reducible graphs.

The research to be discussed in this talk was carried out jointly with Astrid an Huef, Marcelo Laca and Aidan Sims.

Mikael Rørdam (University of Copenhagen, Denmark)

Just infinite groups and C-algebras*

A (discrete) group is called just infinite if it is infinite and all its non-trivial normal subgroups have finite index. There is a well-established theory for just infinite groups, and there are interesting examples of just infinite groups (including, for example, the Grigorchuk groups). In a similar way one can define a (unital) C*-algebra to be just infinite if it is infinite dimensional and all its proper quotients are finite dimensional. Infinite dimensional simple C*-algebras and essential extensions of simple C*-algebras by finite dimensional C*-algebras are just infinite (for trivial reasons). We show that there exist residually finite dimensional just infinite C*-algebras (that can be chosen to be AF-algebras), and we explain some structure results for just infinite C*-algebras. The construction of a just infinite residually finite dimensional AF-algebra can be done using an old result by Bratteli and Elliott which

says that each totally disconnected spectral space arises as the primitive ideal space of an AF-algebra. We discuss possible connections to just infinite groups.

This is work in progress joint with R. Grigorchuk and M. Musat.

Aidan Sims (University of Wollongong, Australia)

KMS states and von Neumann algebras from higher-rank graphs

In 1984, Enomoto, Fujii and Watatani studied KMS states for the gauge action on the Cuntz-Krieger algebra of a $\{0,1\}$-matrix A. They showed that there is a unique KMS state, occurring at inverse temperature determined by the spectral radius r of A, and that the associated factor is the injective Type $\mathrm{III}_{r^{-p}}$ factor, where p is the period of A. I will outline recent work that describes an extension of Enomoto, Fujii and Watatani's results to the higher-rank Cuntz-Krieger algebras of Robertson and Steger, using the machinery of Kumjian and Pask's higher-rank graphs.

This summarises joint work with an Huef, Laca and Raeburn and with Laca, Larsen, Neshveyev and Webster.

Klaus Thomsen (Aarhus Universitet, Denmark)

KMS weights on graph C^*-algebras

The presentation will describe the structure of KMS weights for the gauge action on the C^*-algebra of a strongly connected infinite graph with at most countably many exits.

Mark Tomforde (University of Houston, USA)

Classification of certain C^*-algebras and algebras using techniques from Symbolic Dynamics

In the subject of symbolic dynamics, the shift spaces of finite type arise as edge shifts of finite directed graphs. The shifts of finite type were used in the 1980s and 1990s to classify simple Cuntz-Krieger C^*-algebras, and moreover, the dynamical systems methods were key ingredients in the proofs. Recently, similar techniques have been used to classify unital graph C^*-algebras (in both the simple and certain non-simple cases) in terms of K-theory. More surprisingly, these techniques have also been used to show that certain simple unital Leavitt path algebras (i.e., the algebraic analogues of graph C^*-algebras) can be classified in terms of algebraic K-theory. In this talk I will summarize the current status of these C^*-algebra and algebra classifications, outlining known results and describing open problems.

Andrew Toms (Purdue University, USA)

Mean dimension and the structure of crossed products

We discuss the potential equivalence of two invariants, one for topological dynamical systems and the other for their associated crossed products. Motivation and examples will be favored over proofs!

Stefaan Vaes (University of Leuven, Belgium)

Representation theory and (co)homology for subfactors, λ-lattices and C^*-tensor categories

I first present a joint work with Sorin Popa in which we develop a representation theory for λ-lattices, arising as standard invariants of subfactors, and for rigid C^*-tensor categories. This includes a definition of their universal C^*-algebra and a systematic account of approximation and rigidity properties for subfactors and tensor categories, like (weak) amenability, the Haagerup property and property (T). Secondly, I will present a joint work with Sorin Popa

and Dimitri Shlyakhtenko in which we define (co)homology for λ-lattices and for rigid C^*-tensor categories and use this to define their L^2-Betti numbers.

Stuart White (University of Glasgow, UK)

Colouring simple C-algebras*

A number of recent advances in the structure theory of simple nuclear C*-algebras can be viewed as finitely coloured versions of von Neumann algebraic properties, both at a conceptual level and increasingly at the level of proofs. I'll discuss a variety of examples of this phenomena.

Wilhelm Winter (Westfälische Wilhelms-Universität Münster, Germany)

QDQ vs. UCT

I will discuss the quasidiagonality question for simple nuclear C*-algebras and its relations to the UCT problem. I will also speculate about possible strategies to approach these questions, at least under additional hypotheses. In particular for strongly self-absorbing C*-algebras the situation is somewhat more transparent, and in this case one can formulate a purely infinite counterpart of QDQ.

Makoto Yamashita (Ochanomizu University, Japan)

Classification problems on the compact quantum groups of Lie type

The quantum groups having the same fusion data as the simple Lie groups sit in the intersection of various approaches to quantum groups. In this talk I report on recent developments on the classification of compact quantum groups in this class through the categorical duality and the Poisson boundary for monoidal categories.

Based on joint works with Sergey Neshveyev.

Joachim Zacharias (University of Glasgow, UK)

Higher dimensional Rokhlin properties for group actions on C-algebras*

The classical Rokhlin Lemma is an approximation result for free integer actions on probability spaces. Topological analogues for automorphisms of C*-algebras have been studied since the 1980s. A few years ago higher dimensional versions of the Rokhlin property have been introduced. It turns out that this is a much more flexible concept closely connected to nuclear dimension and the classification programme. For instance it allows to prove a topological version of the Rokhlin Lemma for actions on finite dimensional spaces. More recently Rokhlin dimension has been extended to actions of residually finite amenable groups with links to coarse geometry. We will survey some of these developments.