

The life and works of GKP (Gert Kjærgård Pedersen)

April 13, 1940 – March 15, 2004

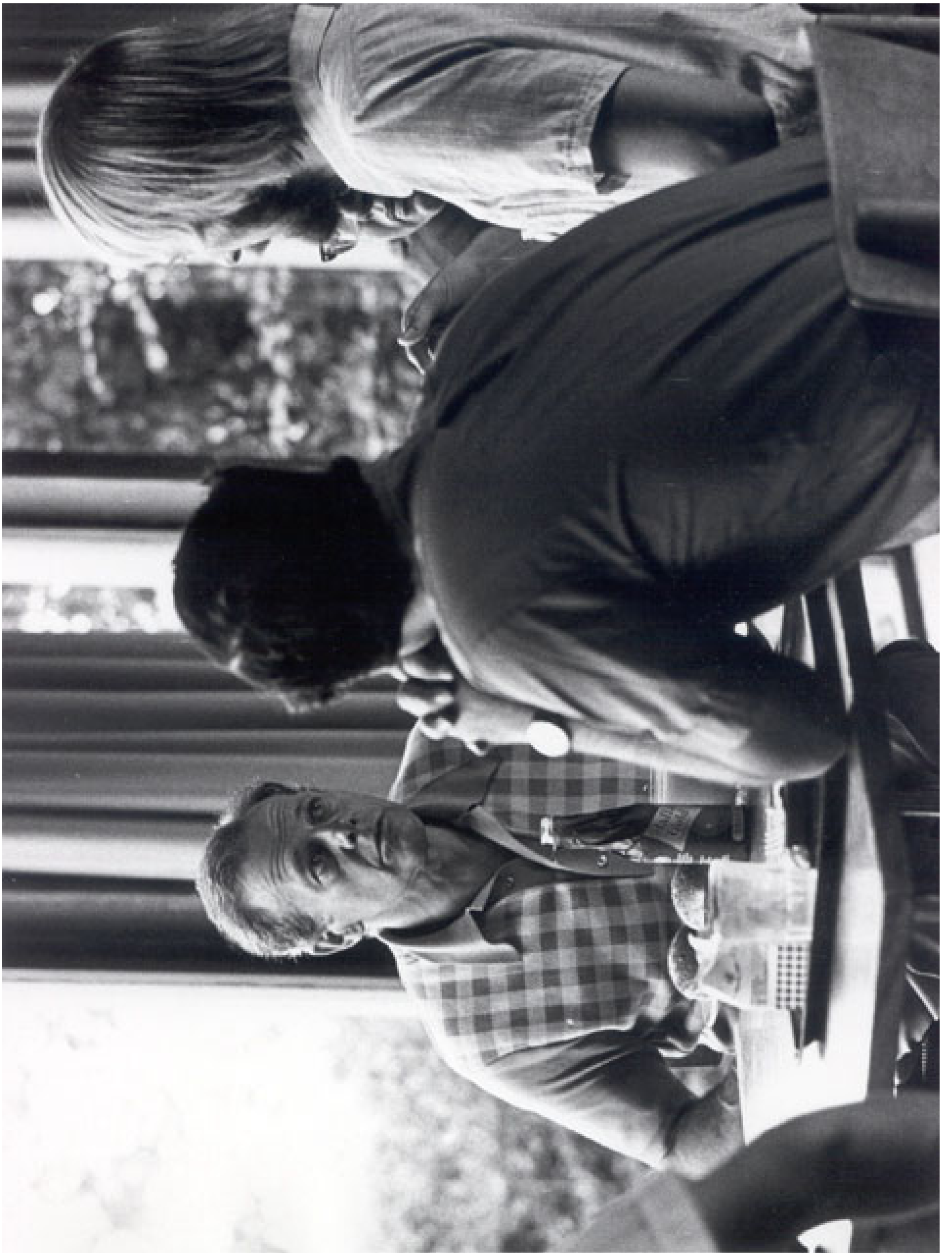
*Dorte Olesen,
The Danish IT-Centre for Education
and Research (UNI•C)*

Early endeavours:

- Member of the Copenhagen Boys Choir 1949 – 1964
- Trips abroad with the Choir, eg Vienna 1963 (Handel's Saul with the Vienna Symphony)
- Learning poetry and languages through songs – and of course regular school
- Built a harpsichord 1966
- Painted a self-portrait and made abstract compositions in the 60es

Entering mathematics

- Math studies University of Copenhagen 1958 – 1964
- Research fellowship 1965 – 1966 spent at University of Aarhus – entered the C^* -family
- The point of departure: C^* -algebras – generalizing notions from topology to the non-commutative setting



Gert K. Pedersen

List of publications

** indicates a survey article*

- [1] Gert K. Pedersen, The structure space of a left ideal, *Mathematica Scandinavica* 14 (1964), 90-92.
- [2] Gert K. Pedersen, Measure theory for C^* algebras, *Mathematica Scandinavica* 19 (1966), 131-145.
- [3] Gert K. Pedersen, Measure theory for C^* algebras II, *Mathematica Scandinavica* 22 (1968), 63-74.
- [4] Gert K. Pedersen, A decomposition theorem for C^* algebras, *Mathematica Scandinavica* 22 (1968), 266-268.
- [5] Gert K. Pedersen, Measure theory for C^* algebras III, *Mathematica Scandinavica* 25 (1969), 71-93.
- [6] Gert K. Pedersen, On weak and monotone σ -closures of C^* algebras, *Communications in mathematical Physics* 11 (1969), 221-226.
- [7] Gert K. Pedersen, Measure theory for C^* algebras IV, *Mathematica Scandinavica* 25 (1969), 121-127.
- [8] Gert K. Pedersen & Nils H. Petersen, Ideals in a C^* algebra, *Mathematica Scandinavica* 27 (1970), 193-204.
- [9] Richard V. Kadison & Gert K. Pedersen, Equivalence in operator algebras, *Mathematica Scandinavica* 27 (1970), 205-222.
- [10] Gert K. Pedersen, Atomic and diffuse functionals on a C^* algebra, *Pacific Journal of Mathematics* 37 (1971), 795-800.

The thesis: C^* -integrals: an approach
to non-commutative measure theory

*You haven't understood a theorem before you
can generalize it*

*The Danish tax administration does not know
the notion of a differentiable function*

Solving 2 problems posed by Kadison in Baton Rouge:

- The Up-Down Theorem: For any C^* -algebra A of operators on a separable Hilbert space there is, for each self-adjoint operator x in the strong closure of A , a sequence (x_n) of self-adjoint operators, each of which is the strong limit of a monotone increasing sequence of self-adjoint operators from A , such that (x_n) is monotone decreasing and strongly convergent to x .
- Suppose A is a C^* -algebra of operators on a Hilbert space such that each maximal abelian C^* -subalgebra of A is weakly closed. Then A is weakly closed.

<http://spicerack.sr.unh.edu/~liming/gpots02.dvi>

A fruitful trip to the US – UCLA and Penn 1971–1972: weights entering the picture

** indicates a survey article*

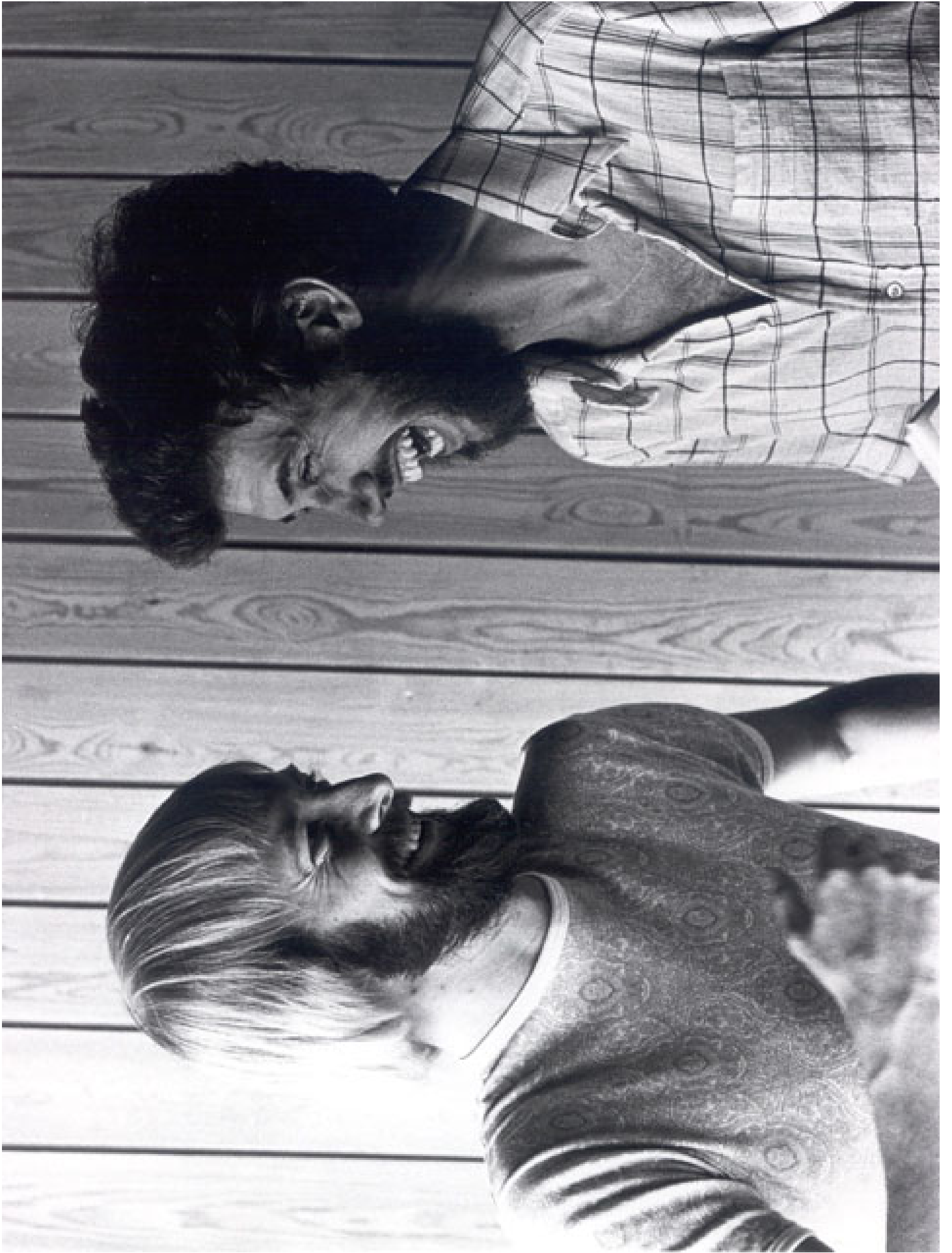
- [11] Gert K. Pedersen, The "Up-Down" problem for operator algebras, Proceedings of the National Academy of Science (USA) 68 (1971), 1896-1897.
- [12] Gert K. Pedersen, Monotone closures in operator algebras, American Journal of Mathematics 94 (1972), 955-962.
- [13] Gert K. Pedersen, Applications of weak* semicontinuity in C^* algebra theory, Duke Mathematical Journal 39 (1972), 431-450.
- [14*] Gert K. Pedersen, C^* integrals, an approach to non-commutative measure theory, Thesis, University of Copenhagen, 1972.
- [15] Gert K. Pedersen, Some operator monotone functions, Proceedings of the American Mathematical Society 36 (1972), 309-310.
- [16] Gert K. Pedersen & Masamichi Takesaki, The operator equation $THT = K$, Proceedings of the American Mathematical Society 36 (1972), 311-312.
- [17] Gert K. Pedersen & Masamichi Takesaki, The Radon-Nikodym theorem for von Neumann algebras, Acta Mathematica 130 (1973), 53-87.
- [18] Gert K. Pedersen, Operator algebras with weakly closed abelian subalgebras, Bulletin of the London Mathematical Society 4 (1972), 171-175.
- [19] Gert K. Pedersen & Erling Størmer, Automorphisms and equivalence in von Neumann algebras II, Indiana University Mathematical Journal 23 (1973), 121-129.

Back in Copenhagen 1972–1974: Multipliers and derivations – and a very special summer school

** indicates a survey article*

- [20] Charles A. Akemann, Gert K. Pedersen & Jun Tomiyama), Multipliers of C^* algebras, *Journal of Functional Analysis* 13 (1973), 277-0301.
- [21] Dorte Olesen & Gert K. Pedersen, Derivations of C^* algebras have semi-continuous generators, *Pacific Journal of Mathematics* 53 (1974), 563-572.
- [22] Charles A. Akemann & Gert K. Pedersen, Complications of semicontinuity in C^* algebra theory, *Duke Mathematical Journal* 40 (1973), 785-795.
- [23] Charles A. Akemann, George A. Elliott, Gert K. Pedersen & Jun Tomiyama), Derivations and multipliers of C^* algebras, *American Journal of Mathematics* 98 (1976), 679-708.
- [24] Gert K. Pedersen, Borel structure in operator algebras, *Det Kongelige Danske Videnskabernes Selskabs Matematisk-Fysiske Meddelelser* 39 Nr. 5 (1974).
- [25*] Gert K. Pedersen, Derivations of operator algebras, C^* Algebras and their Applications to Statistical Mechanics and Quantum Field Theory, 139-146, Editor D. Kastler, North Holland, Amsterdam, 1976.





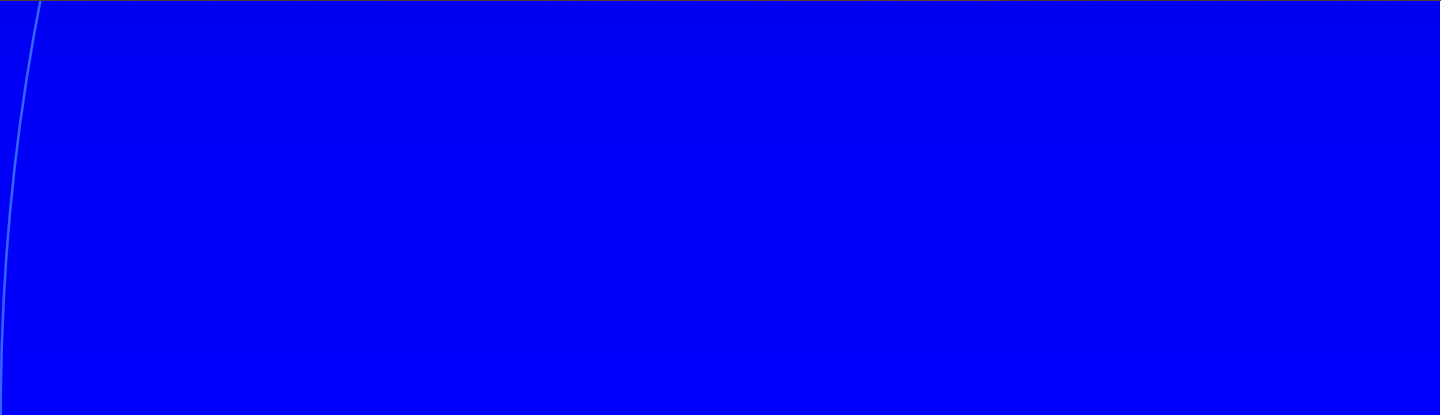
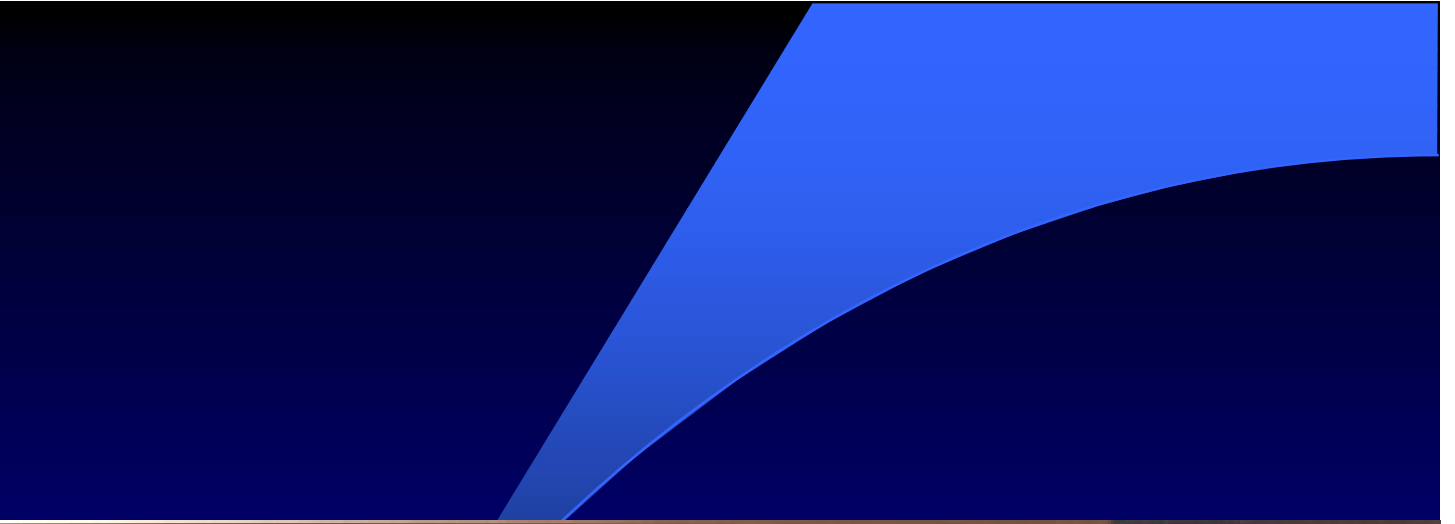
Pursuing the French connection: visits to Marseille & Bandol 1973-84

Inspiration from mathematical physics to study group actions and derivations on C^* -algebras using spectral subspace techniques led among other results to the solution of the open problem concerning lifting derivations:

- Lifting Theorem: If $\pi: A \rightarrow B$ is a surjective morphism between separable C^* -algebras, then for each derivation δ of B there is a derivation $\overline{\delta}$ of A such that $\pi(\overline{\delta}(a)) = \delta(\pi(a))$ for each a in A .

Joint work of the Olesen-Pedersen gang – from the C^* -dynamical workshop at Grants Allé

- [26] Gert K. Pedersen, A non-commutative version of Souslin's theorem, *Bulletin of the London Mathematical Society* 8 (1976), 87-90.
- [27] Gert K. Pedersen, The trace in seminite von Neumann algebras, *Mathematica Scandinavica* 37 (1975), 142-144.
- [28] Gert K. Pedersen, On the operator equation $HT + TH = 2K$, Indiana University *Mathematical Journal* 25 (1976), 1029-1033.
- [29] Gert K. Pedersen, Lifting derivations from quotients of separable C^* algebras, *Proceedings of the National Academy of Science (USA)* 73 (1976), 141-1415.
- [30] Gert K. Pedersen, Lifting groups of automorphisms, *Symposia Mathematica* 20 (1976), Academic Press, London, 161-167.
- [31] Dorte Olesen & Gert K. Pedersen, Groups of automorphisms with spectrum condition and the lifting problem, *Communications in mathematical Physics* 51 (1976), 85-95.
- [32] Gert K. Pedersen, Isomorphisms of UHF algebras, *Journal of Functional Analysis* 30 (1978), 1-16.
- [33] Gert K. Pedersen, Spectral formulas in quotient C^* algebras, *Mathematisches Zeitschrift* 148 (1976), 299-300.
- [34] Dorte Olesen, Gert K. Pedersen & Erling Størmer, Compact abelian groups of automorphisms of simple C algebras, *Inventiones Mathematicae* 39 (1977), 55-64.



Applications of the Connes spectrum to C^* -dynamical systems and some non-mathematical endeavours – with C^* -Pedersen in the middle (1980).

Occasionally, other members of the C^ -family joined in – eg Størmer, Takesaki, Landstad - especially when studying actions of compact abelian groups, where the spectral subspaces became much simpler ; -)*

In fact, the very first Olesen-Pedersen paper – that derivations have semi-continuous generators – was a basic building block when GKP proved the lifting theorem.

- [35] Charles A. Akemann & Gert K. Pedersen, Ideal perturbations of elements in C^* algebras, *Mathematica Scandinavica* 41 (1977), 117-139.
- [36] Dorte Olesen & Gert K. Pedersen, Applications of the Connes spectrum to C^* dynamical systems, *Journal of Functional Analysis* 30 (1978), 179-197.
- [37] Charles A. Akemann & Gert K. Pedersen, Central sequences and inner derivations of separable C algebras, *American Journal of Mathematics* 101 (1979), 1047-1061.
- [38] Gert K. Pedersen, Approximating derivations on ideals of C^* algebras, *Inventiones Mathematicae* 45 (1978), 299-305.
- [39] Joachim Cuntz & Gert K. Pedersen, Equivalence and traces on C^* algebras, *Journal of Functional Analysis* 33 (1979), 135-164.
- [40] Dorte Olesen & Gert K. Pedersen, Some C^* dynamical systems with a single KMS state, *Mathematica Scandinavica* 42 (1978), 111-118.
- [41] Gert K. Pedersen, C^* Algebras and their Automorphism Groups, London Mathematical Society Monographs 14, Academic Press, London/New York, 1979.
- [42] Magnus B. Landstad, Dorte Olesen & Gert K. Pedersen, Towards a Galois theory for C^* crossed products, *Mathematica Scandinavica* 43 (1978), 311-321.
- [43] Joachim Cuntz & Gert K. Pedersen, Equivalence and KMS states on periodic C^* dynamical systems, *Journal of Functional Analysis* 34 (1979), 79-86.
- [44] Gert K. Pedersen, Remarks on the Connes spectrum for C^* dynamical systems, *Journal of Operator Theory* 3 (1980), 143-148.

- [45] Gert K. Pedersen, Čebyšev subspaces of C^* -algebras, *Mathematica Scandinavica* 45 (1979), 147-156.
- [46] Dorte Olesen & Gert K. Pedersen, Applications of the Connes spectrum to C^* dynamical systems, II, *Journal of Functional Analysis* 36 (1980), 18-32.
- [47] Gert K. Pedersen & Hiroshi Takai, Crossed product of C^* -algebras by approximately uniformly continuous actions, *Mathematica Scandinavica* 45 (1979), 282-288.
- [48] Dorte Olesen & Gert K. Pedersen, On a certain C^* crossed product inside a W^* crossed product, *Proceedings of the American Mathematical Society* 79 (1980), 587-590.
- [49] Dorte Olesen, Gert K. Pedersen & Masamichi Takesaki, Ergodic actions of compact abelian groups, *Journal of Operator Theory* 3 (1980), 237-269.
- [50] Gert K. Pedersen, The linear span of projections in simple C^* algebras, *Journal of operator Theory* 4 (1980), 289-296.
- [51] Gert K. Pedersen & Erling Størmer, Traces on Jordan algebras, *Canadian Journal of Mathematics* 34 (1982), 370-373.
- [52] Charles A. Akemann, Joel Anderson & Gert K. Pedersen, Triangle inequalities in operator algebras, *Linear and Multilinear Algebra* 11 (1982), 167-178.
- [53*] Gert K. Pedersen, Dynamical systems and crossed products, *Proceedings of Symposia in Pure Mathematics*, vol. 38 part 1, American Mathematical Society, Providence, R.I., 1982, pp. 27-283.
- [54] Dorte Olesen & Gert K. Pedersen, Applications of the Connes spectrum to C^* dynamical systems, III, *Journal of Functional Analysis* 45 (1982), 357-390.

Getting together with the extended C^* -family - the year at MSRI 1984-85

- [55] Frank Hansen & Gert K. Pedersen, Jensen's inequality for operators and Löwner's theorem, *Mathematische Annalen* 258 (1982), 229-241.
- [56] Erik Christensen & Gert K. Pedersen, Properly infinite AW algebras are monotone sequentially complete, *Bulletin of the London Mathematical Society* 16 (1984), 407-410.
- [57] Karsten Grove & Gert K. Pedersen, Sub-Stonian spaces and corona sets, *Journal of Functional Analysis* 56 (1984), 124-143.
- [58] Karsten Grove & Gert K. Pedersen, Diagonalizing matrices over $C(X)$, *Journal of Functional Analysis* 59 (1984), 65-89.
- [59] Richard V. Kadison & Gert K. Pedersen, Means and convex combinations of unitary operators, *Mathematica Scandinavica* 57 (1985), 249-266.
- [60] Gert K. Pedersen, SAW algebras and corona C^* algebras, contributions to non-commutative topology, *Journal of Operator Theory* 15 (1986), 15-32.
- [61] Gert K. Pedersen, Multipliers of AW algebras, *Mathematisches Zeitschrift* 187 (1984), 23-24.
- [62] Catherine L. Olsen & Gert K. Pedersen, Convex combinations of unitary operators in von Neumann algebras, *Journal of Functional Analysis* 66 (1986), 365-380.
- [63] Dorte Olesen & Gert K. Pedersen, Partially inner C^* dynamical systems, *Journal of Functional Analysis* 66 (1986), 262-281.
- [64] Charles A. Akemann, Joel Anderson & Gert K. Pedersen, Excising states of C^* algebras, *Canadian Journal of Mathematics* 38 (1986), 1239-1260.

***Back in Copenhagen in the late 80es –
the collection of instruments crowned***



Real rank 0 – a natural and numerous class of C^* -algebras – a definition appearing in the much quoted paper with Larry Brown from 1991

- [65] Charles A. Akemann, Joel Anderson & Gert K. Pedersen, Diffuse sequences and perfect C^* algebras, *Transactions of the American Mathematical Society* 298 (1986), 747-762.
- [66] Charles A. Akemann, Joel Anderson & Gert K. Pedersen, Approaching infinity in C^* algebras, *Journal of Operator Theory* 21 (1989), 255-271.
- [67] Gert K. Pedersen, Extreme n-tuples of elements in C^* algebras, *Bulletin of the London Mathematical Society* 19 (1987), 264-270.
- [68] Gert K. Pedersen, Unitary extensions and polar decompositions in a C^* algebra, *Journal of Operator Theory* 17 (1987), 357-364.
- [69] Catherine L. Olsen & Gert K. Pedersen, Corona C^* algebras with applications to lifting problems, *Mathematica Scandinavica* 64 (1989), 63-86.
- [70] Gert K. Pedersen, *Analysis Now*, Graduate Texts in Mathematics 118, Springer Verlag, Berlin, 1988 (revised printing 1995).
- [71] Gert K. Pedersen, Three quavers on unitary elements in C^* algebras, *Pacific Journal of Mathematics* 137 (1989), 169-179.
- [72] Gert K. Pedersen, The corona construction, *Operator Theory: Proceedings of the 1988 GOTS-Wabash Conference*, 49-92, Editors J. B. Conway & B. B. Morrell, Pitman Research Notes in Mathematics 225, Longman Sci. & Tech., UK, 1990.
- [73] Lawrence G. Brown & Gert K. Pedersen, C^* algebras of real rank zero, *Journal of Functional Analysis* 99 (1991), 131-149.
- [74*] Gert K. Pedersen, Real rank of C^* algebras, *Current Topics In Operator Algebras*, 25-32, Editors H. Araki et al., World Scientific Publication Co., Singapore, 1991.

Affine geometry of the unit ball and extensions of C^* -algebras – a recurring interest

- [75] Charles A. Akemann & Gert K. Pedersen, Facial structure in operator algebra theory, Proceedings of the London Mathematical Society 64 (1992), 418-448.
- [76] Gert K. Pedersen, A commutator inequality, Operator Algebras, Mathematical Physics and Low Dimensional Topology, 233-235, Editors R. H. Herman & B. Tanbay, Research Notes in Mathematics 5, A. K. Peters, Mass. USA, 1993.
- [77] Gert K. Pedersen, The λ -function in operator algebras, Journal of Operator Theory 26 (1991), 345-381.
- [78] Lawrence G. Brown & Gert K. Pedersen, On the geometry of the unit ball in a C^* -algebra, Journal für die reine und angewandte Mathematik 469 (1995), 113-147.
- [79] Terry A. Loring & Gert K. Pedersen, Projectivity, transitivity and AF -telescopes, Transactions of the American Mathematical Society 350 (1998), 4313-4339.
- [80] Terry A. Loring & Gert K. Pedersen, Corona extendibility and asymptotic multiplicativity, K -Theory 11 (1997), 83-102.
- [81] Frank Hansen & Gert K. Pedersen, Perturbation formulas for traces on C^* algebras, Publications of the Research Institute for Mathematical Sciences. Kyoto University 31 (1995), 169-178.
- [82] Lawrence G. Brown & Gert K. Pedersen, Approximation and convex decomposition by extremals in a C^* algebra, Mathematica Scandinavica 81 (1997), 69-85.
- [83] Terry A. Loring & Gert K. Pedersen, Smoothing techniques in C^* algebra theory, Journal of Operator Theory 37 (1997), 3-21.
- [84] Gert K. Pedersen, A strict version of the non-commutative Urysohn lemma, Proceedings of the American Mathematical Society 125 (1997), 2657-2660.

Further extensions

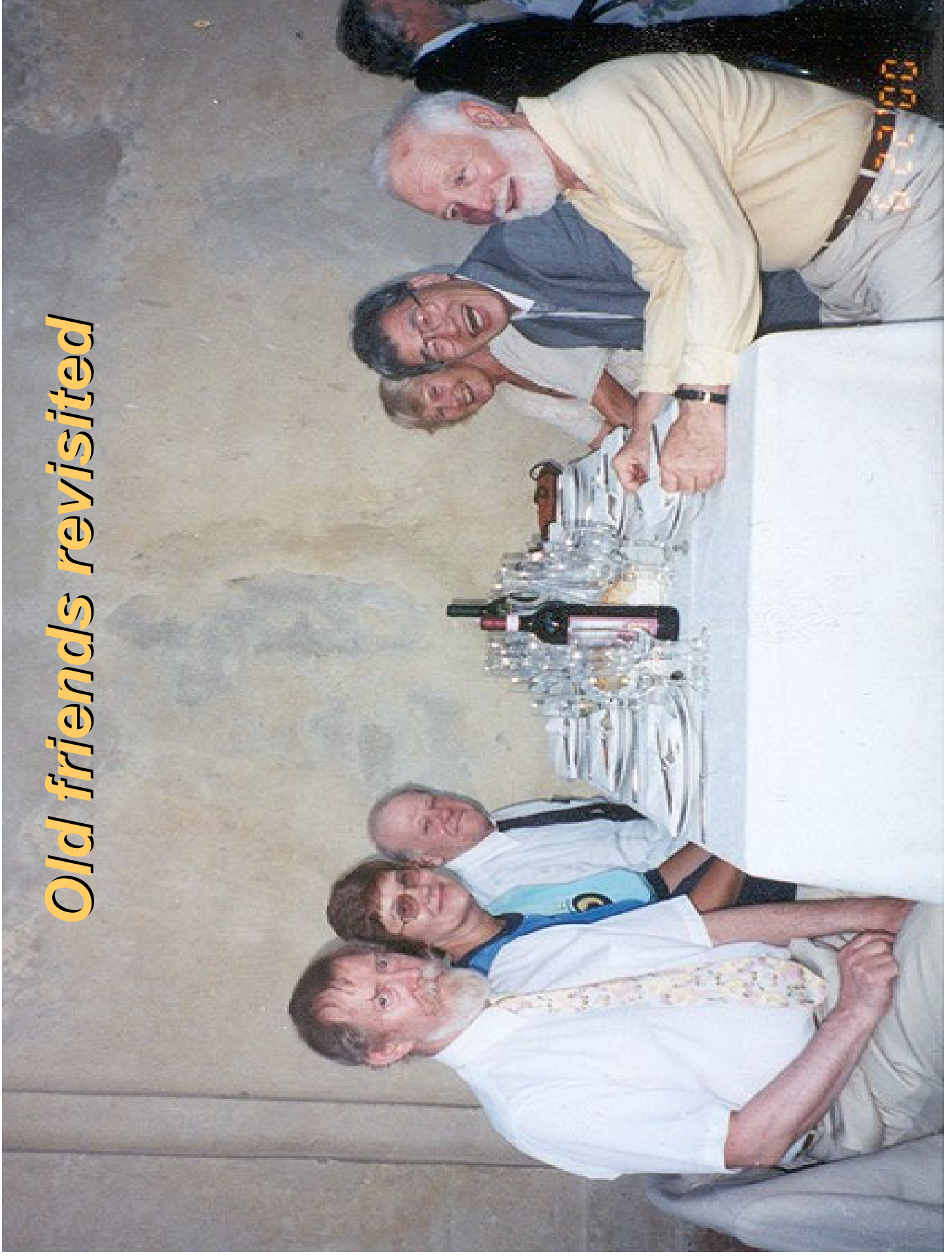
- [85] Søren Eilers, Terry A. Loring & Gert K. Pedersen, Morphisms of extensions of C^* algebras: Pushing forward the Busby invariant, *Advances in Mathematics* 147 (1999), 74-109.
- [86] Søren Eilers, Terry A. Loring & Gert K. Pedersen, Stability of anticommutation relations: An application of noncommutative CW complexes, *Journal für die reine und angewandte Mathematik* 499 (1998), 101-143.
- [87] Søren Eilers, Terry A. Loring & Gert K. Pedersen, Quasidiagonal extensions and *AF*-algebras, *Mathematische Annalen* 311 (1998), 233-249.
- [88] Søren Eilers, Terry A. Loring & Gert K. Pedersen, Fragility of subhomogeneous C^* algebras with one-dimensional spectrum, *Bulletin of the London Mathematical Society* 31 (1998), 337-344.
- [89*] Gert K. Pedersen, Extensions of C^* algebras, *Operator Algebras and Quantum Field Theory*, 2-35, Editors S. Doplicher et al., International Press, Cambridge, Mass., 1997.
- [90*] Gert K. Pedersen, Factorization in C^* algebras, *Expositiones Mathematicae* 16 (1998), 145-156.
- [91] Gert K. Pedersen, Pullback and pushout constructions in C^* algebra theory, *Journal of Functional Analysis* 167 (1999), 243-344.
- [92] Gert K. Pedersen, Operator differentiable functions, *Publications of the Research Institute for Mathematical Sciences. Kyoto University* 36 (2000), 139-157.
- [93] Lawrence G. Brown & Gert K. Pedersen, Extremal K theory and index for C^* algebras, *K-Theory* 20 (2000), 201-241.



Temporarily giving up the star – the Catalan connection - pure algebra moves in

- [94] Pere Ara, Gert K. Pedersen & Francesc Perera, An infinite analogue of rings with stable rank one, *Journal of Algebra* 230 (2000), 608-655.
- [95*] Gert K. Pedersen, Multipliers of C^* algebras, *Encyclopedia of Mathematics*, Kluwer, Dordrecht, Netherlands, to appear.
- [96] Pere Ara, Gert K. Pedersen & Francesc Perera, A closure operation in rings, *International Journal of Mathematics* 12 (2001), 791-812.
- [97*] Gert K. Pedersen, The existence and uniqueness of the Haar integral on a locally compact topological group, *Report 2000*.
- [98] Gert K. Pedersen, Convex trace functions of several variables on C^* algebras, *Journal of Operator Theory*, to appear.
- [99] Pere Ara, Gert K. Pedersen & Francesc Perera, Extensions and pullbacks of QB -rings, *Journal of Algebra and Representation Theory*, submitted.

Old friends revisited



The final papers

- [100] Elliott H. Lieb & Gert K. Pedersen, Convex multivariable trace functions, *Reviews in Mathematical Physics*, 14 (2002), 631-648.
- [101] Lawrence G. Brown & Gert K. Pedersen, Ideal structure in C^* algebras pertaining to low ranks and dimensions, Preliminary preprint 2000.
- [102] Frank Hansen & Gert K. Pedersen, Jensen's operator inequality, *Journal of the London Mathematical Society*, to appear.
- [103] Frank Hansen & Gert K. Pedersen, Jensen's trace inequality in several variables, Preliminary preprint 2002.
- [104] Lawrence G. Brown & Gert K. Pedersen, Interpolation by projections in C^* algebras, Preliminary preprint 2002.
- [105] Lawrence G. Brown & Gert K. Pedersen, Limits in C^* algebras pertaining to low ranks and dimensions, Preliminary preprint 2002.
- [106] Lawrence G. Brown & Gert K. Pedersen, Ideal theory in C^* -algebras pertaining to dimensions and low ranks, Preliminary preprint 2002.
- [107] Gert K. Pedersen, A note on fixed points of completely positive maps, Preprint 2003.
- [108] Gert K. Pedersen & Francesc Perera, Invers limits of rings and multiplier rings, *Proc. Cambridge Philosophical Society* to appear.
- [109] Lawrence G. Brown & Gert K. Pedersen, Non-stable K -theory and extremally rich C^* algebras, Preliminary versions 1992 & 1996; final version in preparation.

Teaching mathematics – 2 exclusive textbooks as the basis for the Copenhagen School of Operator Algebras:

C-algebras and their automorphism groups (1979)*

Analysis NOW (1988, 2nd edition 1995)

Both used in a number of courses at the University of Copenhagen and in fact also elsewhere – and it is always fruitful to have talented students spot minor flaws ; -)

Students supervised for the cand.scient. degree

Niels Holger Petersen, October 1969

Uffe Haagerup, July 1973

Ryszard Nest, February 1975

Niels Erik Wegge-Olsen, September 1988

Ridha Ghabarou, August 1991

Anne Nielsen, September 1992

Søren Eilers, October 1992

Thomas Gauguin Houghton-Larsen, August 1993

Nadia Slavila Larsen, August 1994

Karin Hansen, August 1994

Bent Jacobsen, July 1995

Tina Johansen, September 1996

Claus Dahl, September 1997

Johannes Aastrup, July 1998

Majken Kildegård Andersen, August 1998

Students supervised for the PhD degree

Frank Hansen, October 1983:

Thesis: Convex and monotone matrix functions and their use in operator theory.

Ryszard Nest, January 1987:

Thesis: Cyclic cohomology

Søren Eilers, October 1995:

Thesis: Invariants for AD-algebras

Thomas Gauguin Houghton-Larsen, March 1998:

Thesis: An initial object in the metacategory of homology representations

Nadia Slavila Larsen, June 2000:

Thesis: C^* -algebra crossed products with semigroups

Some of the Principal Contributions to Science- GKP in 1996

- G.K. Pedersen, *The "Up-Down" problem for operator algebras*, Proceedings of the National Academy of Science (USA) **68** (1971), 1896-1897.
- G.K. Pedersen, *Monotone closures in operator algebras*, American Journal of Mathematics **94** (1972), 955-962.
- G.K. Pedersen and M. Takesaki, *The Radon-Nikodym theorem for von Neumann algebras*, Acta Mathematica **130** (1973), 53-87.
- G.K. Pedersen, *Operator algebras with weakly closed abelian subalgebras*, Bulletin of the London mathematical Society **4** (1972), 171-175.
- G.K. Pedersen, *Lifting derivations from quotients of separable C^* -algebras*, Proceedings of the National Academy of Science (USA) **73** (1976), 1414-1415.
- G.K. Pedersen, *Isomorphisms of UHF algebras*, Journal of Functional Analysis **30** (1978), 1-16.
- D. Olesen, G.K. Pedersen and E. Størmer, *Compact abelian groups of automorphisms of simple C^* -algebras*, Inventiones Mathematica **39** (1977), 55-64.

- G.K. Pedersen, *Approximating derivations on ideals of C^* -algebras*, *Inventiones Mathematica* **45** (1978), 299-305.
- D. Olesen and G.K. Pedersen, *Applications of the Connes spectrum to C^* -dynamical systems, III*, *Journal of Functional Analysis* **45** (1982), 357-390.
- R.V. Kadison and G.K. Pedersen, *Means and convex combinations of unitary operators*, *Mathematica Scandinavica* **57** (1985), 249-266.
- L.G. Brown and G.K. Pedersen, *C^* -algebras of real rank zero*, *Journal of Functional Analysis* **99** (1991), 131-149.
- L.G. Brown and G.K. Pedersen, *On the geometry of the unit ball in a C^* -algebra*, *Journal für die reine und angewandte Mathematik* **469** (1995), 113-147.