

The Abel Symposium 2013

Complex Geometry

Trondheim, July 2-5

Program

Tuesday July 2

08:50-09:00 Opening of the Symposium by Trondheim Mayor, Rita Ottervik.

09:00-09:40 Bo Berndtsson: *The openness problem and complex Brunn-Minkowski inequalities*

Coffee

10:00-10:40 Xiang-Yu Zhou: *Some results on L^2 extension problem with optimal estimate*

11:00-11:40 Stephen Yau: *Non-constant CR morphisms between compact strongly pseudo-convex CR manifolds and etale covering between resolutions of isolated singularities*

Lunch

14:00-14:40 Eric Bedford: *Automorphisms of blowups of projective space*

Coffee

15:00-15:40 Tetsuo Ueda: *Semi-parabolic fixed points and their bifurcations in complex dimension 2*

16:00-16:40 Curtis McMullen: *Entropy and dynamics on complex surfaces*



NTNU – Trondheim
Norwegian University of
Science and Technology



ABEL
PRIZE

Wednesday July 3

09:00-09:40 Stefan Nemirovski: *Topology and Several Complex Variables*

Coffee

10:00-10:40 Joël Merker: *Siu-Yeung holomorphic sections of $\text{Sym}^m T_X^*$*

11:00-11:40 Berit Stensønes: *Real analytic domains and plurisubharmonic functions*

Lunch

14:00-14:40 Shing-Tung Yau: *Period Integrals, Counting Curves and Mirror Symmetry*

Coffee

15:00-15:40 Joseph Kohn: *Weakly Pseudoconvex CR Manifolds*

16:00-16:40 Sai-Kee Yeung: *Complex hyperbolicity on the moduli of some higher dimensional manifolds*

19:30 *Banquet on the occasion of Y.T. Siu's 70th birthday*

Thursday July 4

09:00-09:40 Tien-Cuong Dinh: *Positive closed (p,p) -currents and applications in complex dynamics*

Coffee

10:00-10:40 Nessim Sibony: *Dynamics of foliations by Riemann surfaces*

11:00-11:40 Takeo Ohsawa: *Levi flats in Hopf surfaces*

Coffee

12:00-12:40 Zbigniew Błocki: *Hörmander's $\bar{\partial}$ -estimate, some generalizations and new applications*

Lunch

Friday July 5

09:00-09:40 Franc Forstnerič: *Complex analysis and the Calabi-Yau problem*

Coffee

10:00-10:40 Xiaojun Huang: *Analyticity of the local hull of holomorphy for a codimension two real-submanifold in C^n*

11:00-11:40 Peter Ebenfelt: *Partial rigidity of degenerate CR embeddings into spheres*

Lunch

14:00-14:40 Jean-Pierre Demailly: *On the cohomology of pseudoeffective line bundles*

Coffee

15:00-15:40 Samuel Grushevsky: *Meromorphic differentials with real periods, and the geometry of the moduli spaces of Riemann surfaces*

16:00-16:40 Ngaiming Mok: *On the Zariski closure of an infinite number of totally geodesic subvarieties of Ω/Γ*

Abstracts

The openness problem and complex Brunn-Minkowski inequalities

Bo Berndtsson

Tuesday July 2, 09:00–09:40

The openness conjecture of Demailly-Kollár says that if u is a plurisubharmonic function, then the set of all real numbers t such that e^{-tu} is locally integrable near a certain point, is open. I will give a proof of this and discuss the relation to complex Brunn-Minkowski inequalities.

Some results on L^2 extension problem with optimal estimate

Xiang-Yu Zhou

Tuesday July 2, 10:00–10:40

In this talk, we'll present some recent results about L^2 extension problem with optimal estimate, and give some applications.

Non-constant CR morphisms between compact strongly pseudo-convex CR manifolds and etale covering between resolutions of isolated singularities

Stephen Yau

Tuesday July 2, 11:00–11:40

Strongly pseudoconvex CR manifolds are boundaries of Stein varieties with isolated normal singularities. We prove that any non-constant CR morphism between two $(2n-1)$ -dimensional strongly pseudoconvex CR manifolds lying in a n -dimensional Stein variety with isolated singularities are necessarily a CR biholomorphism. As a corollary, we prove that any non-constant self map of $(2n-1)$ -dimensional strongly pseudoconvex CR manifold is a CR automorphism. We also prove that a finite etale covering map between two resolutions of isolated normal singularities must be an isomorphism. This is a joint work with YU-CHAO TU and HUIQING ZUO

Automorphisms of blowups of projective space

Eric Bedford

Tuesday July 2, 14:00–14:40

We will discuss the existence of automorphisms and pseudo-automorphisms of complex manifolds which have positive entropy.

Semi-parabolic fixed points and their bifurcations in complex dimension 2

Tetsuo Ueda

Tuesday July 2, 15:00–15:40

I will talk about bifurcations of semi-parabolic fixed points in dimension 2. The intrinsic structure of a semi-attracting fixed point is investigated and this is used to explain the discontinuity of (filled) Julia sets for the Family of Hénon mappings. This talk is based on a joint work with Eric Bedford and John Smillie.

Entropy and dynamics on complex surfaces

Curtis McMullen

Tuesday July 2, 16:00–16:40

The log of Lehmer's number – a degree 10 algebraic integer, approximately 1.17628 – provides a lower bound on the entropy for all automorphisms of compact complex surfaces.

We will discuss explicit constructions of automorphisms with minimal entropy and, more generally, the synthesis of *projective* K3 surfaces from small Salem numbers, using algebraic number theory, glue groups, integer linear programming and the Torelli theorem.

Topology and Several Complex Variables

Stefan Nemirovski

Wednesday July 3, 09:00–09:40

The talk will be a survey of what is known about the topology of holomorphically, rationally, and polynomially convex domains.

Siu-Yeung holomorphic sections of $\text{Sym}^m T_X^*$

Joël Merker

Wednesday July 3, 10:00–10:40

An n -dimensional complete intersection complex projective algebraic $X^n \subset \mathbb{P}^{n+c}$ of codimension $c \geq n$ larger than its dimension is known to have a wealth of high order symmetric differentials that are everywhere holomorphic (no poles!), although such a (limited) cohomological knowledge happens to be ineffective. Drawing a guided inspiration from Siu and Yeung's seminal 1996 hyperbolicity paper, I will present a construction in which Geometry, Algebra and Combinatorics share their strengths.

Real analytic domains and plurisubharmonic functions

Berit Stensønes

Wednesday July 3, 11:00–11:40

We study pseudoconvex domains in \mathbb{C}^3 with real analytic boundary. The focus will be on demonstrating how one can use the underlying structure to bump the domain out to type in different directions.

Period Integrals, Counting Curves and Mirror Symmetry

Shing-Tung Yau

Wednesday July 3, 14:00–14:40

Weakly Pseudoconvex CR Manifolds

Joseph Kohn

Wednesday July 3, 15:00–15:40

Let $\Omega \subset \mathbb{C}^n$ be a domain with a smooth pseudoconvex boundary $b\Omega$ which is weakly pseudoconvex at $P \in b\Omega$ (i.e. the Levi form is not positive definite at P). Then the regularity of solutions of $\bar{\partial}\varphi = \alpha$ near P , where φ is a $(0, q)$ -form and α a $(0, q + 1)$ -form, depends on the behavior of the germs of complex analytic varieties of dimension q through P . If U is a neighborhood of P regularity on $U \cap \bar{\Omega}$ is studied by means of subelliptic estimates of the “energy” form

$$Q(\varphi, \varphi) = \|\bar{\partial}\varphi\|^2 + \|\bar{\partial}^*\varphi\|^2,$$

defined on $(q, 0)$ -forms in $C^\infty(\bar{\Omega})$ which are supported in $U \cap \bar{\Omega}$ and belong to the domain of $\bar{\partial}^*$. The subelliptic estimate holds if there exist constants ε and C such that $\|\varphi\|_\varepsilon^2 \leq CQ(\varphi, \varphi)$, for all such forms, where the left hand side denotes the ε Sobolev norm. The D’Angelo q -type of P is the maximum order of contact that a complex q -dimensional variety through P can have with $b\Omega$. The main theorem, due to Catlin, is that a subelliptic estimate holds at P if and only if the D’Angelo type of P is finite. In the case of when $b\Omega$ is real analytic in a neighborhood of P there is an equivalent necessary and sufficient condition for subellipticity, namely that the ideal type of P is finite. This condition is expressed in terms of ideals of subelliptic multipliers, these consist of germs of functions at P . When the ideal q -type is infinite the above subelliptic estimate does not hold. This follows from the fact that infinite ideal q -type of P is equivalent to the existence of a q -dimensional complex analytic variety through P that is contained in $b\Omega$. Here we present an explicit construction of such a variety which gives insight into the relation between the D’Angelo type and the ideal type.

The study of these ideals have a direct application to the study of singularities of complex analytic varieties. Consider the variety $V \subset \mathbb{C}^{n-1}$ given by $h_1(z_1 \dots z_{n-1}) = \dots = h_m(z_1 \dots z_{n-1}) = 0$, where the h_j are holomorphic functions that vanish at the origin. Let $\Omega \subset \mathbb{C}^n$ be a pseudoconvex domain defined by $z \in \Omega \mid r(z) < 0$ where in a neighborhood of the origin we have

$$r(z_1 \dots z_n) = \operatorname{Re}(z_n) + \sum |h_j(z_1 \dots z_{n-1})|^2.$$

The multiplier ideals at the origin are generated by ideals of germs of holomorphic functions which are invariants of V .

Complex hyperbolicity on the moduli of some higher dimensional manifolds

Sai-Kee Yeung

Wednesday July 3, 16:00–16:40

The purpose of the talk is to explain some joint work with Wing-Keung To on the problem of complex hyperbolicity on families of some higher dimensional manifolds.

Positive closed (p,p) -currents and applications in complex dynamics

Tien-Cuong Dinh

Thursday July 4, 09:00–09:40

I will present some recent progress in the study of positive closed currents of arbitrary bi-degree: regularization, super-potential, density and intersection. Several applications to dynamics will be given: entropy estimates, properties of dynamical degrees and equidistribution. The talk is based on joint works with Nessim Sibony.

Dynamics of foliations by Riemann surfaces

Nessim Sibony

Thursday July 4, 10:00–10:40

I will discuss some basic facts for dynamics of foliations (with singularities) by Riemann surfaces. I will emphasize ergodicity results à la Birkhoff and their relation with Nevanlinna theory. The lecture is based on joint works with J.E Fornæss, T.C Dinh and V.A Nguyen.

Levi flats in Hopf surfaces

Takeo Ohsawa

Thursday July 4, 11:00–11:40

A compact Levi flat hypersurface in a complex manifold is said to be of q -concave type if it admits a neighborhood system consisting of q -concave manifolds in the sense of Andreotti-Grauert. The real analytic Levi flat hypersurfaces of 1-concave type in Hopf surfaces are classified.

Hörmander's $\bar{\partial}$ -estimate, some generalizations and new applications

Zbigniew Błocki

Thursday July 4, 12:00–12:40

We will present some new applications of the classical Hörmander's L^2 estimate for the $\bar{\partial}$ equation. Among them the Ohsawa-Takegoshi extension theorem with optimal constant, one-dimensional Suita conjecture, as well as Nazarov's approach to the Bourgain-Milman inequality in convex geometry.

Complex analysis and the Calabi-Yau problem

Franz Forstnerič

Friday July 5, 09:00–09:40

I shall describe how methods of complex analysis can be used to give new results on the conformal Calabi-Yau problem. I will show that every bordered Riemann surface admits a proper complete holomorphic immersion into the ball of \mathbb{C}^2 , and a proper complete embedding as a holomorphic null curve in the ball of \mathbb{C}^3 . Since the real and the imaginary parts of a null curve in \mathbb{C}^3 are conformally immersed minimal surfaces in \mathbb{R}^3 , this gives a bounded complete conformal minimal immersion of any bordered Riemann surface into \mathbb{R}^3 . The main advantage of our methods when compared to the existing ones is that we do not need to change the conformal type of the Riemann surface. (Joint work with A. Alarcon, University of Granada.)

Analyticity of the local hull of holomorphy for a codimension two real-submanifold in \mathbb{C}^n

Xiaojun Huang

Friday July 5, 10:00–10:40

We discuss the formal and analytic flattening for a real submanifold of codimension two in a complex Euclidean space near a CR singular point. This has an immediate application for obtaining the regularity result for the local hull of holomorphy or finding a Levi-flat hypersurface with a prescribed real-analytic boundary.

Partial rigidity of degenerate CR embeddings into spheres

Peter Ebenfelt

Friday July 5, 11:00–11:40

We shall consider degenerate CR embeddings f of a strictly pseudoconvex hypersurface $M \subset \mathbb{C}^{n+1}$ into a sphere \mathbb{S} in a higher dimensional complex space \mathbb{C}^{N+1} . The degeneracy of the mapping f will be characterized in terms of the ranks of the CR second fundamental form and its covariant derivatives. In 2004, the speaker, together with X.

Huang and D. Zaitsev, established a rigidity result for CR embeddings f into spheres in low codimensions. A key step in the proof of this result was to show that degenerate mappings are necessarily contained in a complex plane section of the target sphere (partial rigidity). In the 2004 paper, it was shown that if the total rank d of the second fundamental form and all of its covariant derivatives is $< n$ (here, n is the CR dimension of M), then $f(M)$ is contained in a complex plane of dimension $n + d + 1$. The converse of this statement is also true, as is easy to see. When the total rank d exceeds n , it is no longer true, in general, that $f(M)$ is contained in a complex plane of dimension $n + d + 1$, as can be seen by examples. In this talk, we shall show that (well, explain how) when the ranks of the second fundamental form and its covariant derivatives exceed the CR dimension n , then partial rigidity may still persist, but there is a "defect" k that arises from the ranks exceeding n such that $f(M)$ is only contained in a complex plane of dimension $n + d + k + 1$. Moreover, this defect occurs in general, as is illustrated by examples.

On the cohomology of pseudoeffective line bundles

Jean-Pierre Demailly

Friday July 5, 14:00–14:40

The lecture will present various results concerning the cohomology of pseudoeffective line bundles on compact Kähler manifolds, twisted with corresponding multiplier ideal sheaves. In case the curvature is strictly positive in the sense of currents, the prototype is the well known Nadel vanishing theorem. We are interested here in the case where the curvature is merely semipositive. Various results and applications will be discussed, including a recent vanishing theorem due to Junyan Cao (forthcoming PhD thesis in Grenoble), and a study of simple compact Kähler 3-folds (joint work with F. Campana and M. Verbitsky from April 2013).

Meromorphic differentials with real periods, and the geometry of the moduli spaces of Riemann surfaces

Samuel Grushevsky

Friday July 5, 15:00–15:40

On the Zariski closure of an infinite number of totally geodesic subvarieties of Ω/Γ

Ngaiming Mok

Friday July 5, 16:00–16:40

Let Ω be a bounded symmetric domain, $\Gamma \subset \text{Aut}(\Omega)$ be a torsion-free lattice, $X := \Omega/\Gamma$. Let $Z \subset X$ be an irreducible quasi-projective variety such that Z is the Zariski closure of the union of an infinite family of totally-geodesic complex subvarieties $S_\alpha \subset Z$, $\alpha \in A$. Under a non-degeneracy condition one expects Z to be also totally geodesic, so that Z is again uniformized by a bounded symmetric domain. This set-up is related to a well-known problem on Shimura varieties $X = \Omega/\Gamma$ in which one tries to characterize the Zariski closure of an infinite number of ‘special’ subvarieties S_α . Here special subvarieties are defined by arithmetic conditions, but it is known that they are always totally geodesic. While the case where S_α are 0-dimensional, for which the problem is called the André-Oort Conjecture, cannot be dealt with directly using methods of complex geometry, the case where S_α are of positive dimension is naturally a problem in complex geometry. From our complex-analytic perspective, no arithmeticity assumption is placed on $\Gamma \subset \text{Aut}(\Omega)$, and the ‘distinguished’ subvarieties are simply the totally-geodesic subvarieties of X .

Using methods of Kähler geometry, we solve the afore-mentioned problem in the rank-1 case. A generalization of the argument to bounded symmetric domains Ω leads to the study of holomorphic isometries from complex unit balls B^m to Ω . We explain a method along this line of thoughts for solving the general problem, and illustrate how the problem is solved when Z is a complex surface and $S_\alpha \subset Z$ are totally-geodesic holomorphic curves using our recent results on holomorphic isometries with respect to the Bergman metric.